Spring Block 2

Algebra

Small steps

Small steps

Step 9	Find pairs of values
Step 10	Solve problems with two unknowns

1-step function machines

Notes and guidance

In this small step, children begin to formally look at algebra for the first time by exploring function machines. This builds on their work in earlier years using operations and their inverses to find missing numbers.

Children need to learn the meanings of the terms “input”, “output”, “function” and “rule”. At first, they are given a number, told what to do to it using any of the four operations and calculate the output. They then move on to finding the input from a given output, using inverse operations.

Finally, children explore examples where the input and output are given, but the function is not. They should recognise that one rule may fit for some of the numbers given, but not for all, and that they need to find a rule that works for all the numbers.

Things to look out for

- Children may carry out the function on the output when working out the missing input, rather than using the inverse operation.
- Children may find a function that works for some of the numbers given, but not all.

Key questions

- How does the function machine work?
- What is the difference between an input and an output?
- If you know the input and function, how can you work out the output?
- If you know the output and function, how can you work out the input?
- What is the inverse of ______?
- Does your rule work for all the sets of numbers?

Possible sentence stems

- If the input is ______, the output is ______
- If I know the output, I need to …
- If the input is ______ and the output is ______, then the function is ______

National Curriculum links

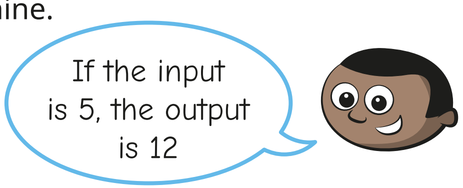
- Use simple formulae
- Generate and describe linear number sequences

1-step function machines

Key learning

- Mo has made a function machine.

input → $+ 7$ → output



  - If the input is 7, what is the output?
  - If the input is 4,023, what is the output?

- Complete the table for the function machine.

input → $- 5$ → output

Input	5	23	5.1	23.2	0	–3	–5
Output							

- Complete the table for the function machine.

input → $\times 4$ → output

Input	3	10	0	2.5	0.25	7	70
Output							

- The function machine shows the output, but not the input.

input → $+ 5$ → output 6

Talk to a partner about how you can work out the input.

- Work out the missing inputs.

input → $\times 3$ → output: 21, 0.6

input → $+ 0.5$ → output: 5.5, 20.6

- What are the missing functions?

input		output
15	→ ? →	5
9	→ ? →	–1
33	→ ? →	23

input		output
15	→ ? →	5
9	→ ? →	3
33	→ ? →	11

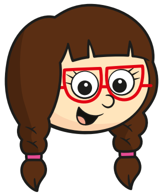
What do you notice?

1-step function machines

Reasoning and problem solving

Jo and Ron are working out the rule for the function machine.

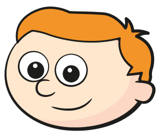
input		output
4	→ →	44



The rule is + 40

Jo

The rule is × 11



Ron

Who do you agree with?

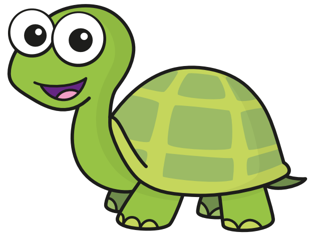
Explain your answer.

Either could be correct.

Tiny is working out the missing number.

Input	9	7	3.5	–2
Output	19	17	13.5	

The missing number is –12



Explain Tiny's mistake.

What is the missing number?

8

2-step function machines

Notes and guidance

In this small step, children move on to explore function machines with two steps.

As with 1-step machines, they start by looking at examples where the input is given and they need to find the output, using a mix of any of the four operations. Discuss why it is important that they follow the order of the functions; for example, the output of × 5 then + 3 will be different from + 3 then × 5

Children then move on to finding the input when the output is known by using the inverse of each function, recognising that they need to start with the second function when working backwards.

Children then look at problems where the input and output are given, but one of the two functions is missing. They may choose to do this problem working forwards or backwards.

Things to look out for

- Children may not follow the order of the functions, and it is important to explore the effect this can have.
- When finding the input, children may do the inverse of the first function first.

Key questions

- Which function should you apply first?
- What happens if you do not follow the functions in the correct order?
- What is the inverse of ______?
- When given the output, which function should you do first?
- What is the input if the output is ______?
- What is the missing function if the input is ______, the output is ____ and one of the functions is ______?
- Does it always matter what order you apply the functions?

Possible sentence stems

- First, I am going to ______, then I am going to ______
- If the input is ______, then the output is ______
- The inverse of ______ then ______ is ______ then ______

National Curriculum links

- Use simple formulae
- Find pairs of numbers that satisfy an equation with two unknowns
- Enumerate possibilities of combinations of two variables

2-step function machines

Key learning

- Here is a 2-step function machine.

input → × 2 → + 1 → output

  - If the input is 5, what is the output?
  - If the input is 10, what is the output?

- Complete the tables for the function machines.

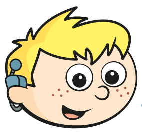
input → + 2 → × 3 → output

Input	3	4	5	10
Output				

input → × 3 → + 2 → output

Input	3	4	5	10
Output				

What do you notice?

- 

I think of a number, double it, then add 4

  - What answer will Max get if he thinks of 20?
  - What number would Max need to think of to get the answer 20?

- Work out the missing inputs.

input → + 10 → − 3 → output

input	output
	8
	23

input → × 4 → + 5 → output

input	output
	13
	45

- What are the missing functions?

input → × 5 → ? → output

input	output
3	16
25	126

input → ? → ÷ 2 → output

input	output
29	10
11	1

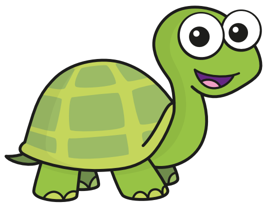
2-step function machines

Reasoning and problem solving

Tiny is using a 2-step function machine.

input			output
	× 4	+ 2	

You can multiply numbers in any order, and you can add numbers in any order. This means you can solve this function machine in any order.

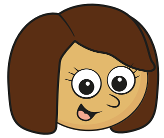


Do you agree with Tiny?

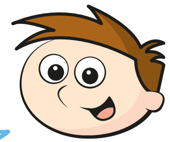
Explain your answer.

No

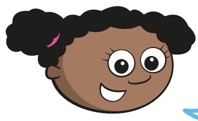
input			output
2			4
10			20



Kim: The missing functions are × 4 and ÷ 2



Teddy: The missing functions are × 2 and × 1



Whitney: There only needs to be one function, which is × 2

Who do you agree with?

What other functions would work?

They are all correct.

Form expressions

Notes and guidance

This small step is children's first experience of forming algebraic expressions using letters to represent numbers.

Children learn that phrases such as "2 more than a number" can be written more simply as, for example, "$x + 2$" or "$y + 2$". They also learn the convention that, for example, "3t" means 3 multiplied by t; as multiplication can represent repeated addition, this is also a simpler way of writing $t + t + t$. They use cubes and base 10 ones to represent expressions, with each cube representing an unknown number, x (or any letter), and the ones representing known numbers.

Children then revisit function machines, where x (or any letter) can represent the input. Discuss why it is not important at this stage to know what x represents, and that it could be any number input into the function machine.

Bar models can also be used to support children's understanding.

Things to look out for

- Children may assume that certain letters always represent specific numbers, for example a means 1, b means 2, c means 3 and so on.
- Children may not see $a \times 3$ and 3a as the same thing.

Key questions

- What could x represent?
- How can you represent this expression using a bar model?
- How else can you write $a + a$?
- What is the same and what is different about the expressions $x + 5$ and $5x$?
- If the input is p, what is the output?
- If m is the input, what is the output after the first operation? What is the output after the second operation?

Possible sentence stems

- ______ more than x can be written as ______
- ______ + ______ + ______ = 3 × ______ = ______
- If I have ______ x and I add/subtract ______ x, then I have ______ x altogether.

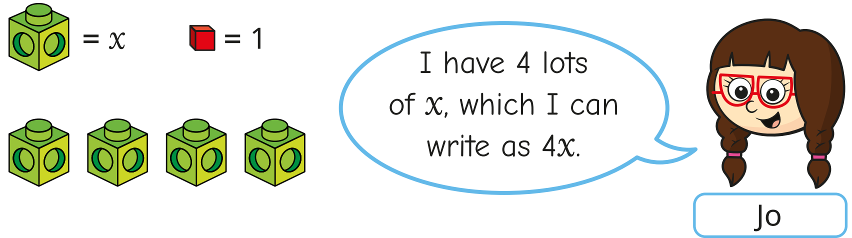
National Curriculum links

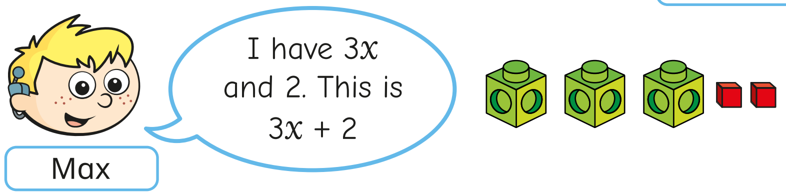
- Use simple formulae
- Express missing number problems algebraically

Form expressions

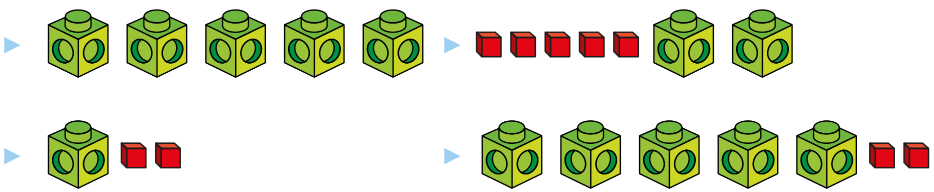
Key learning

- Jo and Max are using cubes to represent unknown numbers and base 10 ones to represent 1





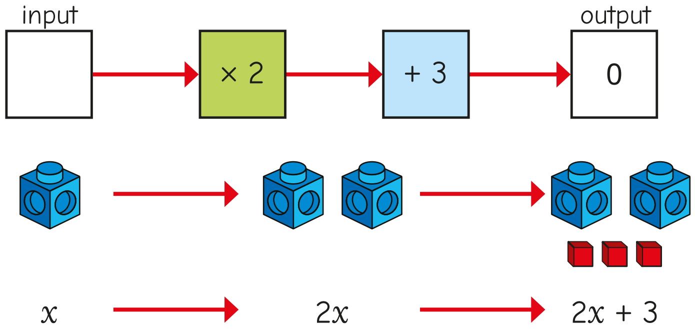
Use Jo and Max's method to write algebraic expressions for each set of cubes and base 10 ones.



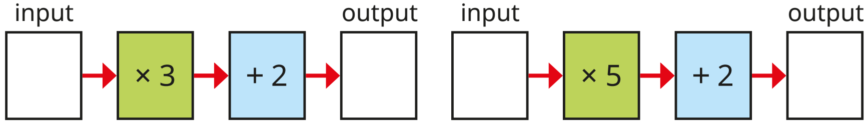
- Use cubes and base 10 to represent the algebraic expressions.

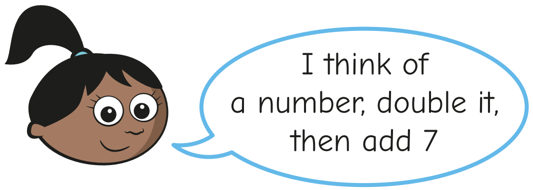
$y + 3$	$2y + 1$	$5 + 5y$

- Dan writes an expression for the 2-step function machine.



Use Dan's method to write an expression for each function machine.



- 

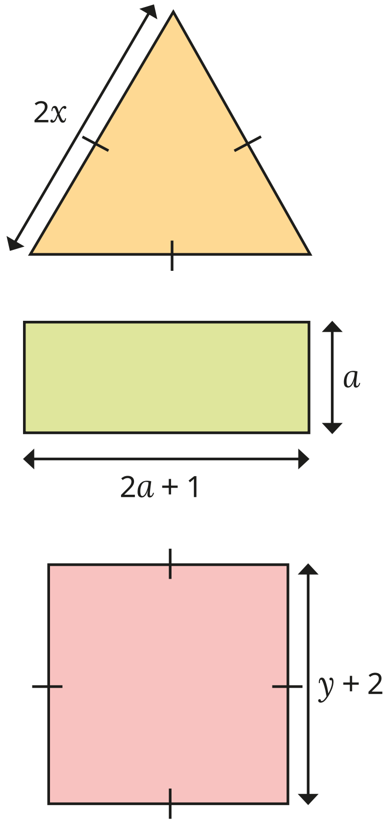
Sam calls the number she first thinks of x.

Write an expression for the number that Sam is thinking of after she has done the two calculations.

Form expressions

Reasoning and problem solving

Write expressions for the perimeters of the shapes.



$6x$

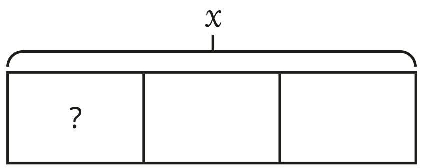
$6a + 2$

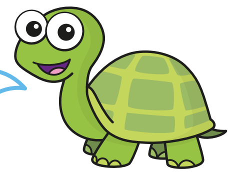
$4y + 8$

The perimeter of a rectangle is $12x$.

What could the sides of the rectangle be?

multiple possible answers, e.g. $5x$ and x





> The bar model represents $3x$ because x is the total and there are three parts.

Do you agree with Tiny?

Explain your answer.

No

Each part is $x \div 3$

Substitution

Notes and guidance

In this small step, children find values of expressions by substituting numbers in place of the letters.

Children should understand that the same expression can have different values depending on what number is substituted into it. Before working with letters, children explore concrete and pictorial representations. By assigning values to, for example, a square and a triangle, they can work out square + triangle. Similarly, building on representations from the previous step, if they assign a value to a cube, they can work out the value of an expression.

Children then move on to substituting numbers into abstract algebraic expressions such as $3a + 1$. This can be linked to the earlier learning of function machines, and thought of as "multiply by 3 and then add 1", or bar models, replacing each occurrence of the letter with its value.

Things to look out for

- Children may think that a is always equal to 1, b always equal to 2 and so on.
- If $a = 3$, children may see $2a$ as 23 rather than $2 \times 3 = 6$
- Children may misinterpret expressions such as $2a + 3$ as $5a$.

Key questions

- If 1 cube is worth ______, what are 3 cubes worth?
- What does $4x$ mean? If you know the value of x, how can you work out the value of $4x$?
- What does "substitute" mean?
- How can you represent the expression as a bar model? Which parts of the bar model can you replace with a number? What is the total value of the bar model?
- Which part of the expression can you work out first? What is the total value of the expression?

Possible sentence stems

- If ______ is worth ______, then ______ is worth ______
- To work out the value of ______, I need to replace the letter ______ with the number ______ and then calculate ______

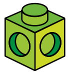
National Curriculum links

- Use simple formulae
- Express missing number problems algebraically

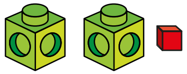
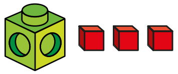
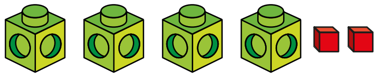
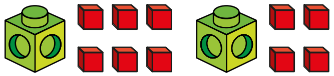
Substitution

Key learning

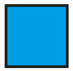
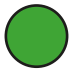
- Ann gives values to these cubes.

   = 5  = 1

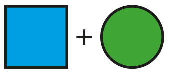
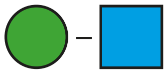
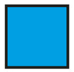
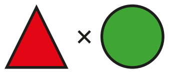
  Work out the values of the sets of cubes.

  - 
  - 
  - 
  - 

- Tom draws three shapes and gives each one a value.

   = 3  = 4  = 5

  Work out the values of the expressions.

  - 
  - 
  - 2 × 
  - 

- Here are three expressions.

$5a$	$a + 5$	$10 - a$

  - Which expression has the greatest value when $a = 1$?
  - Which expression has the greatest value when $a = 5$?
  - Which expression has the greatest value when $a = 10$?

- Esther generates a sequence by substituting $n = 1$, $n = 2$, $n = 3$, $n = 4$ and $n = 5$ into the expression $4n + 1$

  When $n = 1$,

  $4n + 1 = 4 \times 1 + 1 = 4 + 1 = 5$

  Work out the other numbers in Esther's sequence.

  What patterns can you see?

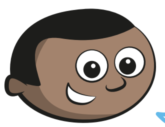
- If $a = 5$ and $b = 12$, work out the values of the expressions.

$a + b$	$b - a$	$2b - a$	$a + 3b$	$b \div 2$	$20 \div a$

Substitution

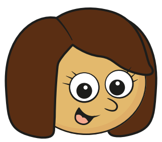
Reasoning and problem solving

$x = 2c + 6$



Is Mo correct?

Explain why.



Explain why Kim is wrong.

What is the correct value of x when $c = 5$?

No

Kim has put the 2 next to the 5 to make 25, instead of multiplying 2 by 5

$x = 16$

Work out the missing values in the table.

x	$3x$	$3x + 5$
1		
3		
	12	
	36	
		20
		26

row 1: 3, 8

row 2: 9, 14

row 3: 4, 17

row 4: 12, 41

row 5: 5, 15

row 6: 7, 21

Find the value of c when $a = 10$

$p = 2a + 5$

$c = 10 - p$

$c = -15$

Formulae

Notes and guidance

In this small step, children are introduced to formulae using symbols for the first time, although they will be familiar with the idea of a formula in words, for example area of a rectangle = length × width.

Building on the previous steps, children substitute into formulae to work out values, noticing the effect that changing the input has on the output. Looking at familiar relationships between two or more variables will help to develop children's understanding, for example the number of days in a given number of weeks, the number of legs on a given number of insects and so on.

Children should recognise the difference between a formula and an expression, noticing that an expression does not have the equals sign, but a formula does.

Things to look out for

- Children may mix up the variables in a formula, for example using $w = 7d$ to represent the formula for the number of days in a given number of weeks.

Key questions

- What is a formula?
- What formulae do you know?
- How is a formula similar to/different from an expression?
- What is the formula for ______?
- If the formula is $t = 3s + 1$ and you know that $s =$ ______, how can you work out t?
- Which letter(s) do you know the value of? Which letter(s) can you work out?

Possible sentence stems

- In the formula ______, the letter ______ represents ______ and the letter ______ represents ______
- To work out ______ when I know ______, I substitute ______ into the formula.

National Curriculum links

- Use simple formulae
- Express missing number problems algebraically

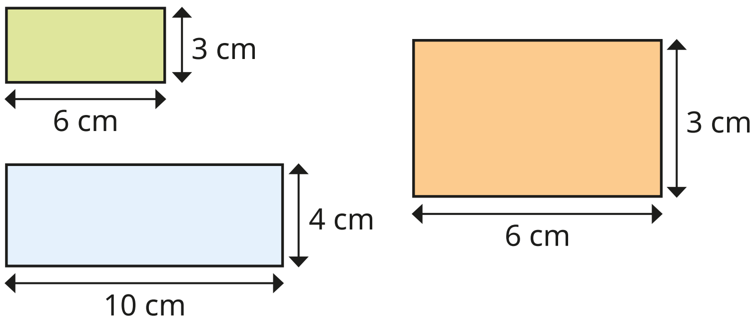
Formulae

Key learning

- Ron uses a formula to work out the areas of rectangles.

  $A = lw$
  When $l = 7$ and $w = 4$, $A = 7 \times 4 = 28$

  - What do the letters A, l and w represent?
  - Use the formula to find the areas of the rectangles.

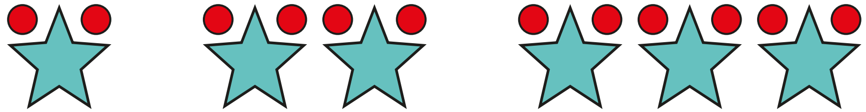


- The time taken to cook a turkey is 90 minutes, plus an additional 20 minutes for every kilogram of turkey.

  This can be written as the formula $T = 90 + 20m$

  - What do the letters T and m represent?
  - Use the formula to work out the time to cook:
    - a 3 kg turkey
    - a 10 kg turkey

- Fay makes a sequence of patterns with stars and circles.



  Complete the table to show the number of circles and stars in the patterns.

Number of stars	1	2	3	5		
Number of circles	2				18	30

  If s = number of stars and c = number of circles, which formula describes Fay's pattern?

  $s = 2 + c$ $c = s + 2$ $c = 2s$ $s = 2c$ $2s = c + 2$

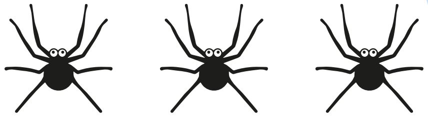
- The table shows the total number of legs on a given number of ants.

Number of ants (a)	1	2	3		
Number of legs (L)	6			30	72

  Complete the table and write a formula that describes the pattern.

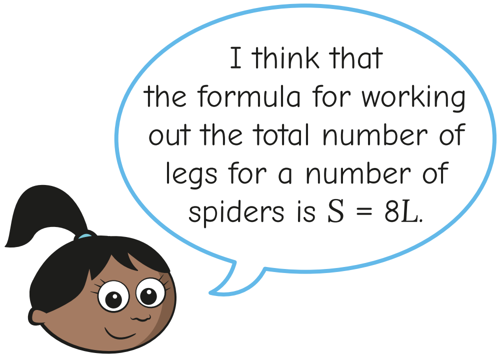
Formulae

Reasoning and problem solving



S = number of spiders

L = total number of legs



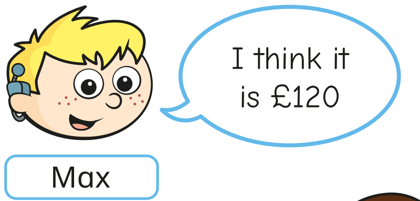
Do you agree with Sam?

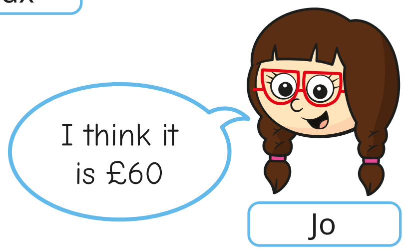
Explain your answer.

No

Max and Jo use this formula to work out the cost in pounds (C) of four hours (h) of cleaning.

$$C = 20 + 10h$$





Who do you agree with?

Explain your answer.

Jo

Form equations

Notes and guidance

In this small step, children form equations from diagrams and word descriptions.

Begin the step by looking at the difference between an algebraic expression and an equation. An expression, such as $2x + 6$, changes value depending on the value of x, whereas in an equation, such as $2x + 6 = 14$, x has a specific value. You may need to remind children of the algebraic conventions learnt earlier in the block, for example writing $a + a + a$ (or $a \times 3$) as $3a$ and "4 more than b" as $b + 4$

Various representations can be used to support children's understanding, including bar models, part-whole models and cubes and counters with a designated value. It is important that children understand that, for example, the letter c represents the numerical value of the cube rather than the cube itself.

Things to look out for

- Children may look to work out the value rather than represent the information as an equation.
- Children may make errors using algebraic notation, for example confusing $3x$ and $x + 3$

Key questions

- If a is a number, how do you write "3 times the value of a"?
- How do you write "4 more than the number x"?
- If 4 more than the number x is equal to 26, how can you write this as an equation?
- Is an equation the same as or different from a formula?
- What is the difference between an equation and an expression?
- Can you write the equation a different way?
- Is ______ an equation or an expression? How do you know?

Possible sentence stems

- ______ + ______ +______ = 3 × ______ = ______
- The equation ______ means that the expression ______ is equal to ______
- ______ more/less than ______ is equal to ______ can be written as the equation ______ = ______

National Curriculum links

- Express missing number problems algebraically

Form equations

Key learning

- Tom thinks of a number and calls it x.

  Which expression represents 5 more than Tom's number?

$5x$	$x + 5$	$x - 5$	$x \div 5$

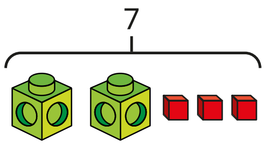
  Double Tom's number is 64

  Which equation shows this information?

$x + 2 = 64$	$x \div 2 = 64$	$2x = 64$	$x - 2 = 64$

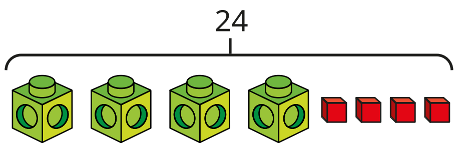
- Max has represented some equations.

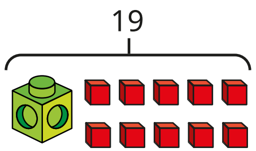
  Each linking cube represents y and each base 10 cube represents 1



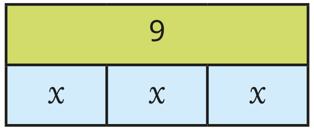
$2y + 3 = 7$

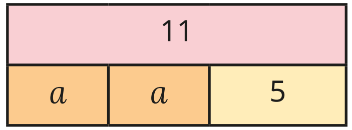
  What equations are represented?

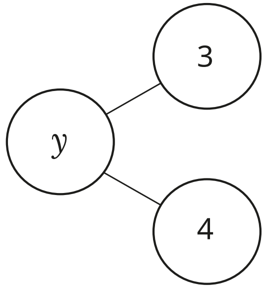


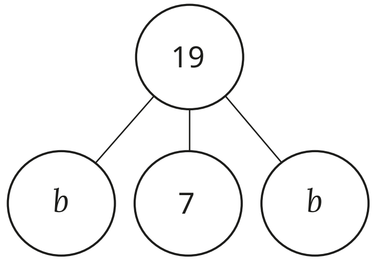


- Write equations to match the models.









- A book costs £5 and a magazine costs £n.

  The total cost of the book and the magazine is £8

  Write this information as an equation.

- Write algebraic equations for the word problems.
  - I think of a number and subtract 17. My answer is 20
  - I think of a number and multiply it by 5. My answer is 45

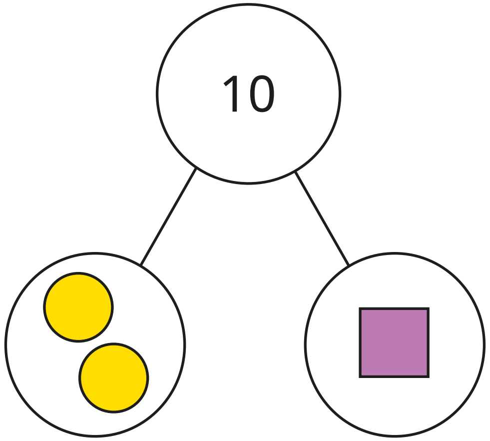
- Draw bar models to represent the equations.

$x + 5 = 11$	$2y = 15$	$3a + 9 = 30$

Form equations

Reasoning and problem solving

Here is a part-whole model.



Write an equation representing the part-whole model.

Each shape has a different integer value.

What values might the shapes have?

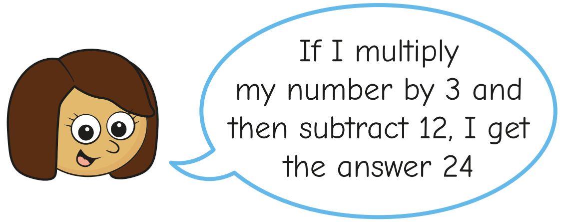
Using c for circles and s for squares:

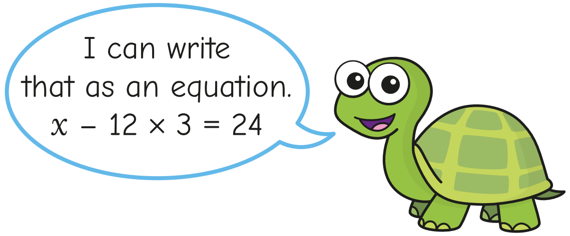
$2c + s = 10$

multiple possible answers, e.g.

$c = 2$, $s = 6$

Kim is thinking of a number.





What mistake has Tiny made?

Write the correct equation for Kim's problem.

Tiny has not applied the operations in the correct order.

$3x - 12 = 24$

Solve 1-step equations

Notes and guidance

In this small step, children look at solving equations formally for the first time. At first, they might find the notation a bit confusing, but encourage them to consider equations as a different way of writing "missing number" problems.
For example, $x + 5 = 12$ is the same as ______ + 5 = 12

It is useful to begin by looking at "think of a number" questions, such as "Mo thinks of a number, adds 7 and gets the answer 20. What was his original number?" and relating this to the equation $n + 7 = 20$. Similarly, you can build on earlier learning using function machines, relating finding an input for a given output to solving the corresponding equation. In both cases, children should see that using inverse operations helps to solve the equations.

Things to look out for

- Children may not use the inverse operation to solve an equation, for example $x + 3 = 5$, so $x = 8$
- Children may think that the values of letters are permanently fixed. For example, having solved an equation for x, they may apply this value for x to a different equation.

Key questions

- What does the expression $3x$ mean?
- If you know 3 times the value of a number, how can you use this to work out the number?
- How can you represent the problem as a bar model?
- How can you represent the problem as an equation?
- What is the inverse of ______?
- What does the bar model show?
  What can you use it to work out?
- How can you draw a function machine to represent the equation?
  How does the function machine help you to solve the equation?

Possible sentence stems

- The inverse of ______ is ______
- If ______ has been added to a number to give ______, then to work out the number I need to ______ from ______

National Curriculum links

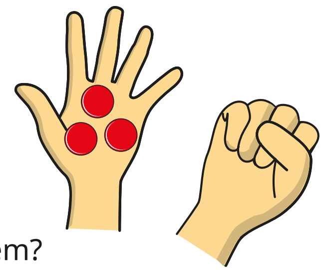
- Express missing number problems algebraically

Solve 1-step equations

Key learning

- Ben has 9 counters altogether.

  He has 3 counters in his left hand and c counters in his closed right hand.

  Which equation represents this problem?

$c - 3 = 9$	$3c = 9$	$c + 3 = 9$	$c = 9 + 3$

  How many counters does he have in his closed hand?

- Fay thinks of a number.

  She adds 9 to her number.

  She gets the answer 15

  What was her original number?

  Explain how the equation $x + 9 = 15$ represents this problem.

- Dan thinks of a number and multiplies it by 3 to get the answer 12

  Which equation shows this?

$3x = 12$	$3 + x = 12$	$x - 3 = 12$	$x \div 3 = 12$

  What was Dan's original number?

- Write expressions for the outputs of the function machines.

  input x → $+ 9$ → output

  input y → $- 3$ → output

  input p → $\times 5$ → output

  input t → $\div 2$ → output

  If the output of all the machines is 20, write and solve equations to find the values of the letters.

- Write an equation to represent each bar model.

  Then find the value of x for each one.

15		
x	x	x

12	
x	7

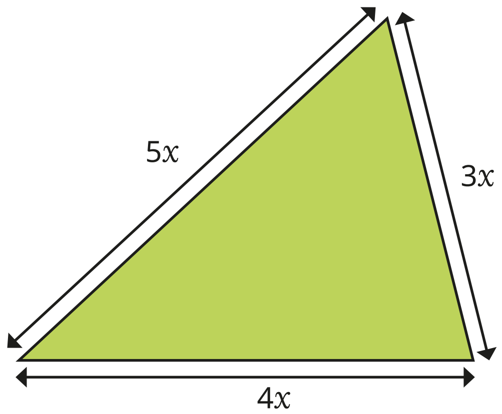
- Solve the equations.

$3x = 21$	$y + 5 = 11$	$z - 6 = 8$	$p \div 3 = 10$

Solve 1-step equations

Reasoning and problem solving

The perimeter of the triangle is 216 cm.



Form an equation to find the value of x.

Work out the lengths of the sides of the triangle.

$12x = 216$

$x = 18$

54 cm, 72 cm and 90 cm

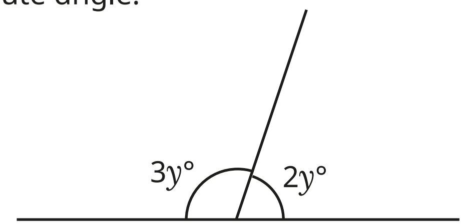
$x - 9 = 0$

Do you agree with Tiny?

Explain your answer.

No

Work out the size of the acute angle.



How can you check your answer?

72°

Solve 2-step equations

Notes and guidance

In this small step, children move on to solving equations with two steps.

As with 1-step equations, initially equations of this type can be represented by 2-step "think of a number" problems and/or function machines, where children work backwards using inverse operations to find the original number or input. They can then link this to finding an unknown in a 2-step equation.

Children can also use concrete resources to represent the problems and to work out missing numbers. Bar models are another useful representation, as they give a visual clue to the steps needed to work out the unknowns. It is useful to have the abstract representation alongside the models to help develop understanding.

Things to look out for

- Children may think the values of letters are permanently fixed. For example, having solved an equation for x, they may apply this value for x to a different equation.
- When "working backwards" to solve equations, children may do the inverse operations in the wrong order.

Key questions

- If you know 3 more than $2x$, how can you work out $2x$?
- If you know 5 less than $2x$, how can you work out $2x$?
- How can you represent the problem with a bar model? Which part(s) of the bar model do you already know? Which part(s) can you work out?
- How can you represent the problem with an equation? What is the first step you need to take to solve the equation?
- How can you represent the equation using a function machine? How can you use the function machine to help you solve the equation?

Possible sentence stems

- If ______ x + ______ = ______, then ______ x = ______, so x = ______
- The first step in solving the equation is to ______
  The second step in solving the equation is to ______

National Curriculum links

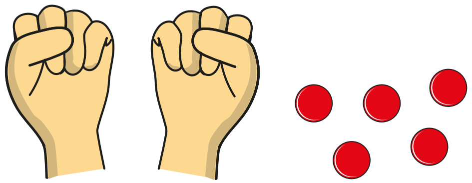
- Express missing number problems algebraically

Solve 2-step equations

Key learning

- Tommy has 17 counters.

  He puts the same number of counters (c) in each hand and has some left over.

  Which equation shows this?

$c + 2 = 5$	$2c = 17$	$2c + 5 = 17$	$2c + 17 = 5$

  Solve the equation to work out how many counters Tommy has in each hand.

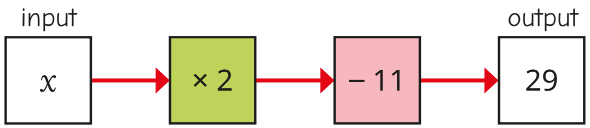
- Kay thinks of a number.

  She multiplies the number by 2 and then adds 5

  She gets the answer 29

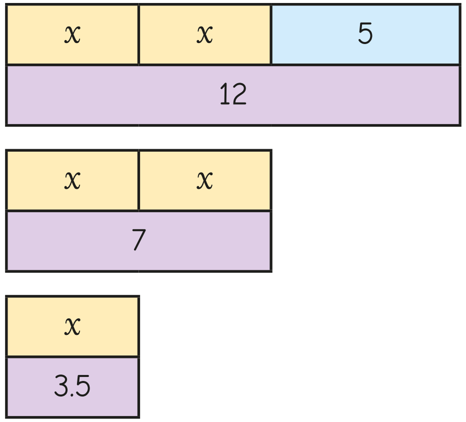
  What number did Kay think of?

- Explain how this 2-step function machine shows the equation $2x - 11 = 29$

  Work out the value of x.

- Ron uses a bar model to solve an equation.

  $2x + 5 = 12$

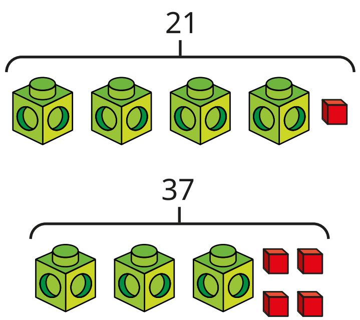
  $2x = 7$

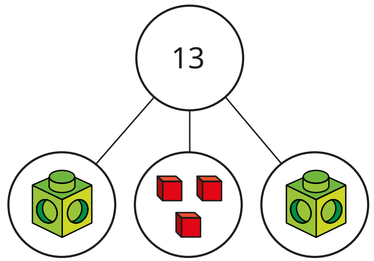
  $x = 3.5$

  Use Ron's method to solve the equations.

$3b + 4 = 19$	$20 = 4b + 2$

- Write and solve equations for the models.

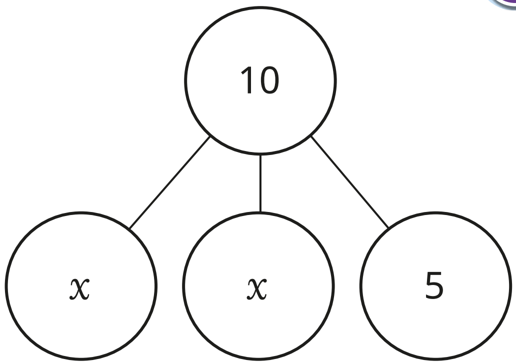
  

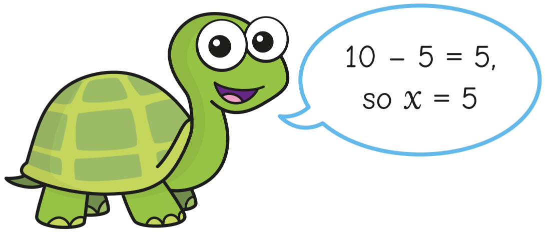
  

Solve 2-step equations

Reasoning and problem solving

Tiny is working out the value of x.



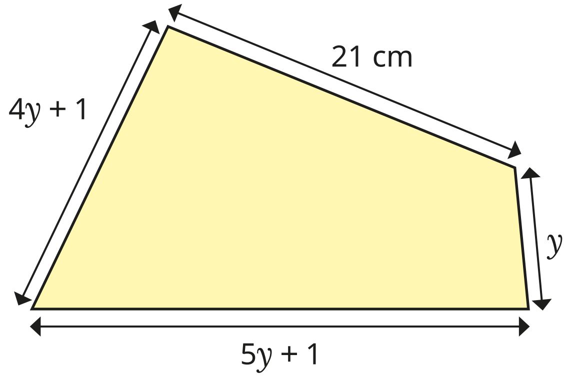


Do you agree with Tiny?

Explain your reasoning.

No

The perimeter of the quadrilateral is 83 cm.



Work out the value of y.

Explain your method to a partner.

y = 6 cm

Find pairs of values

Notes and guidance

In this small step, children explore equations with two unknown values, recognising that these can have several possible solutions.

Children can use substitution to work out pairs of possible values. For example, if $x + y = 9$, they find the values of y for different values of x. They should work systematically to find all the possible integer values. A table is a good way to support this. In this step, the possible values will always be integers greater than or equal to zero, but this could be extended to negative and decimal values. Begin with simple equations of the form $x + y =$ ______ or $ab =$ ______, before moving on to more complex equations that include multiples of the unknowns, for example $2x + 3y =$ ______

It is important that children understand that they cannot know the exact value of the two unkowns, as they do not have enough information.

Things to look out for

- Children may not consider zero as a possible value for one of the unknowns.
- Children may need support to work systematically to find all possible solutions.

Key questions

- What two numbers could add together to make ______?
- What could the values of x and y be in the equation ______?
- Why are there several possible answers for this question?
- Have you found all the possible pairs of values? How do you know?
- In the equation ______, if $x =$ ______, what must the value of y be? If x is a different value, does y also change?
- How can you draw a bar model to represent the equation ______?

Possible sentence stems

- In the equation $x + y =$ ______, if $x =$ ______ then $y =$ ______
- If the product of p and q is ______, then p could be ______ and q could be ______

National Curriculum links

- Find pairs of numbers that satisfy an equation with two unknowns
- Enumerate possibilities of combinations of two variables

Find pairs of values

Key learning

- x and y are both whole numbers.

  $x + y = 5$

  Ann creates a table to work out the possible sets of values of x and y.

x	y	$x + y$
0	5	5
		5
		5
		5
		5
		5

  Work systematically to complete Ann's table.

- a and b are both whole numbers.

  $a \times b = 24$

  Create a table to show all the possible sets of values for a and b.

- p and q are both whole numbers less than 12

  $p - q = 3$

  Find all the possible values of p and q.

- x and y are both whole numbers.

  $x > y$ $x + y = 25$

  - If x is odd and y is even, what are the possible pairs of values for x and y?
  - If x and y are both even, what are the possible pairs of values for x and y?
  - If x is a multiple of 5 and y is even, what are the possible pairs of values for x and y?

  Create your own problem like this for a partner.

- a and b are integers.

  $3a + 2b = 20$

  Work out three possible pairs of values for a and b.

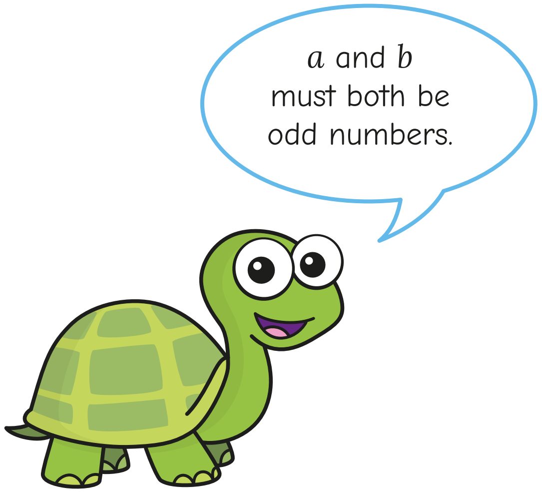
  Compare methods with a partner.

Find pairs of values

Reasoning and problem solving

a and b are both whole numbers.

$$ab + b = 18$$



Is Tiny correct?

Explain your answer.

No

a, b and c are integers between 0 and 5

$a + b = 6$ $b + c = 4$

Find the values of a, b and c.

How many possibilities can you find?

$a = 2, b = 4, c = 0$

$a = 3, b = 3, c = 1$

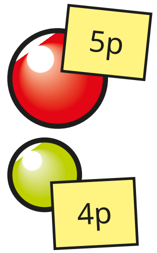
$a = 4, b = 2, c = 2$

$a = 5, b = 1, c = 3$

Large beads costs 5p and small beads cost 4p.

Sam spends 79p on beads.

What possible combinations of large beads (l) and small beads (s) could she buy?



Write each possible combination as an expression.

$3l + 16s$

$7l + 11s$

$11l + 6s$

$15l + s$

Solve problems with two unknowns

Notes and guidance

Building on previous learning, in this small step children solve problems with two unknowns when more than one piece of information is given, so there is only one possible solution.

Examples include the case where the sum and the difference of both unknowns is given. Bar models are used throughout the step to represent problems and to support children's understanding.

Other structures are also explored, including where one of the unknowns is a multiple of the other. In this case, a bar model can be used to work out the values of the numbers if either their total or their difference is known. Finally, children look at equations with two unknowns where the coefficient of only one of the unknowns is different, for example $x + 2y = 17$ and $x + 5y = 38$. Again, a bar model will help children to see why $3y$ must be equal to 21, after which y and x can be found.

Things to look out for

- Children may use trial and error rather than a bar model approach.
- Children may think that there are several possible solutions, as in the last step.

Key questions

- How can you represent this information as a pair of equations?
- How can you represent this information with a bar model?
- What information does the bar model show? What else can you work out?
- How can you draw a bar model to represent the problem? Which parts can you label straight away? What else can you work out?
- Is there more than one possible solution?

Possible sentence stems

- If ______ lots of x is worth ______, then x = ______ ÷ ______ = ______
- If I know the value of ______, I can find the value of ______ by substituting into the equation ______

National Curriculum links

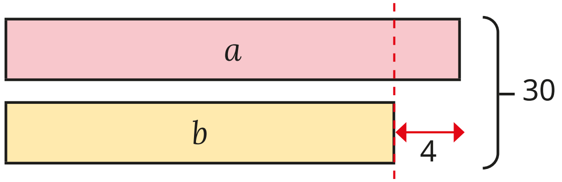
- Express missing number problems algebraically
- Find pairs of numbers that satisfy an equation with two unknowns

Solve problems with two unknowns

Key learning

- The sum of a and b is 30

  The difference between a and b is 4

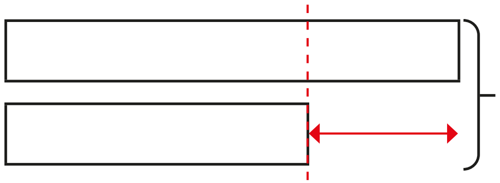
  Use the bar model to work out the values of a and b.

- Here is some information about two numbers, x and y.

  $x + y = 10$

  $x - y = 2$

  - Label the information on the bar model.

  - Use the bar model to work out the values of x and y.

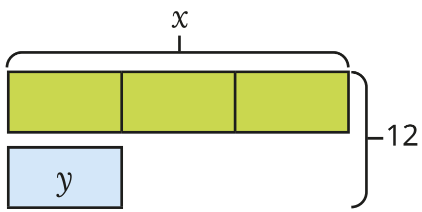
- The sum of two numbers, p and q, is 55

  The difference between p and q is 7

  Show this as a bar model and find the values of p and q.

- The sum of x and y is 12

  x is 3 times the size of y.

  - Explain how you can use the bar model to work out the value of y.
  - What is the value of x?

    Are there any other possible solutions?

- The sum of two numbers, a and b, is 18

  a is one-fifth the size of b.

  Draw a bar model to represent this problem and work out the values of a and b.

- Tom and Ann both go for a walk.

  Between them they walk 21 km.

  Tom walks 6 times as far as Ann does.

  How much further does Tom walk than Ann?

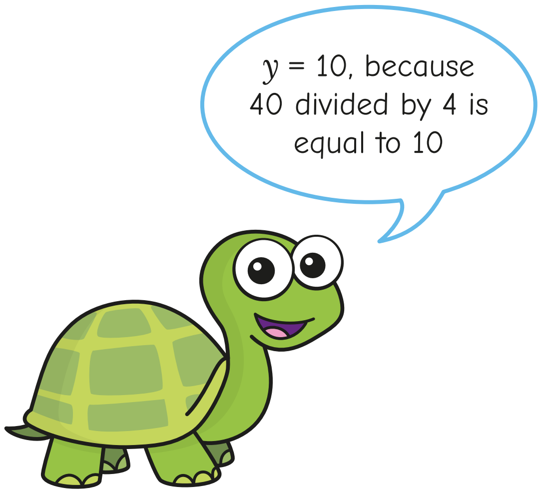
Solve problems with two unknowns

Reasoning and problem solving

The sum of x and y is 40

x is 4 times the size of y.

What is the value of y?



Show that Tiny is wrong.

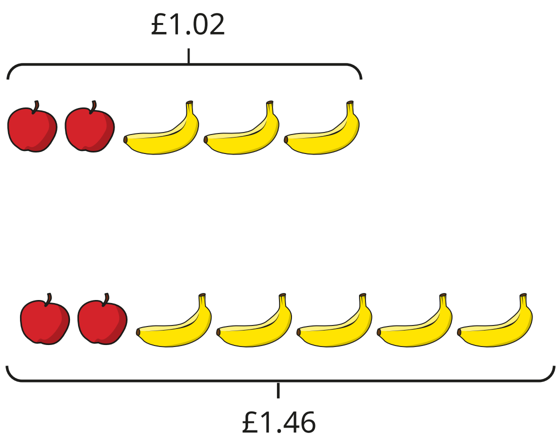
Find the correct values of x and y.

If y = 10, x = 40 and $x + y$ = 50

y = 8 and x = 32

Two apples and three bananas cost £1.02

Two apples and five bananas cost £1.46



What is the total cost of one apple and one banana?

40p

Spring Block 3

Decimals

Small steps

Step 1	Place value within 1
Step 2	Place value – integers and decimals
Step 3	Round decimals
Step 4	Add and subtract decimals
Step 5	Multiply by 10, 100 and 1,000
Step 6	Divide by 10, 100 and 1,000
Step 7	Multiply decimals by integers
Step 8	Divide decimals by integers

Small steps

Step 9	Multiply and divide decimals in context

Place value within 1

Notes and guidance

Children encountered numbers with up to 3 decimal places for the first time in Year 5. This understanding is recapped in this small step and built upon in the rest of the block.

Children represent numbers with up to 3 decimal places using counters and place value charts, identify the values of the digits in a decimal number and partition decimal numbers in a range of ways.

Children know the relationship between the different place value columns, for example hundredths are 10 times the size of thousandths and one-tenth the size of tenths.

In this step, numbers are kept within 1 to allow children to focus on the value of the decimal places. In the next step, they explore numbers greater than 1 with up to 3 decimal places.

Things to look out for

- Children may confuse the words “thousand” and “thousandth”, “hundred” and “hundredth”, and “ten” and “tenth”.
- Children may use the incorrect number of placeholders, and so write the incorrect number.

Key questions

- What does each digit in a decimal number represent? How do you know?
- How many tenths/hundredths/thousandths are there in 1 whole?
- How many thousandths are there in 1 hundredth?
- What is the value of the digit ______ in the number ______?
- Which is greater, 0.3 or 0.14? How do you know?

Possible sentence stems

- There are ______ tenths, ______ hundredths and ______ thousandths.

  The number is ______
- There are ______ in ______
- ______ is 10 times/one-tenth the size of ______

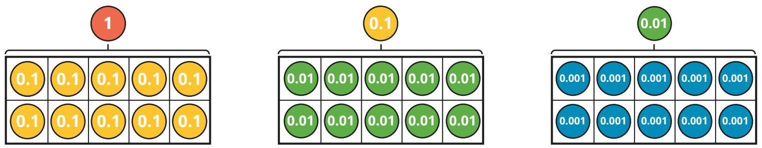
National Curriculum links

- Identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places

Place value within 1

Key learning

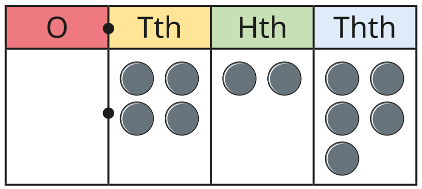
- Use the diagrams to complete the sentences in as many ways as possible.



_______ is one-tenth the size of _______

_______ is 10 times the size of _______

- Scott has made a number on a place value chart.



Complete the sentences to describe Scott's number.

There are _______ ones, _______ tenths, _______ hundredths and _______ thousandths.

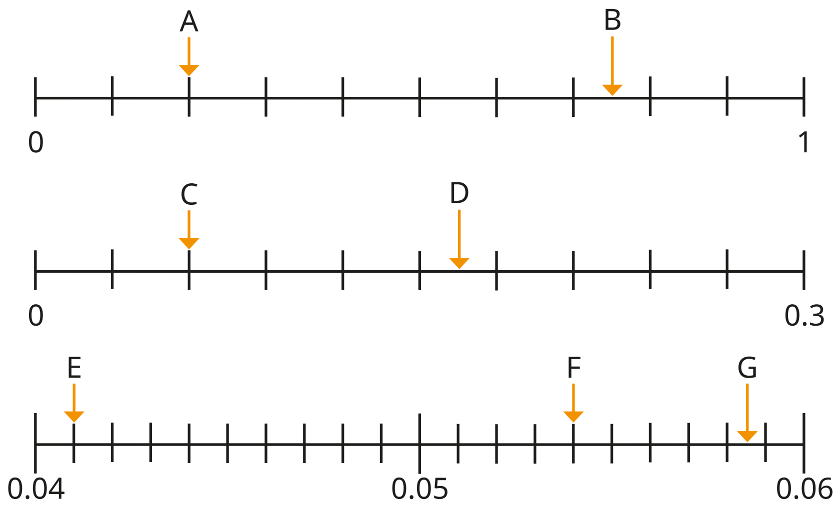
The number is _______

- Use a place value chart and plain counters to represent the numbers.

0.613	0.84	0.402	0.051

- Complete the number sentences.
  - 0.2 + 0.06 + 0.009 = _______
  - 0.4 + _______ + 0.001 = 0.451
  - _______ = 0.006 + 0.1 + 0.03
  - 0.6 + 0.003 = _______

- What decimal numbers are the arrows pointing to?



- Ron has partitioned 0.536

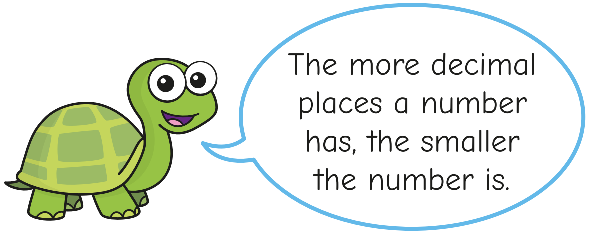
0.536 = 0.4 + 0.13 + 0.006

Use a place value chart to partition 0.536 a different way.

Compare answers with a partner.

Place value within 1

Reasoning and problem solving



Do you agree with Tiny?

Explain your answer.

No

O	Tth	Hth	Thth

Use four counters to make a number less than 1

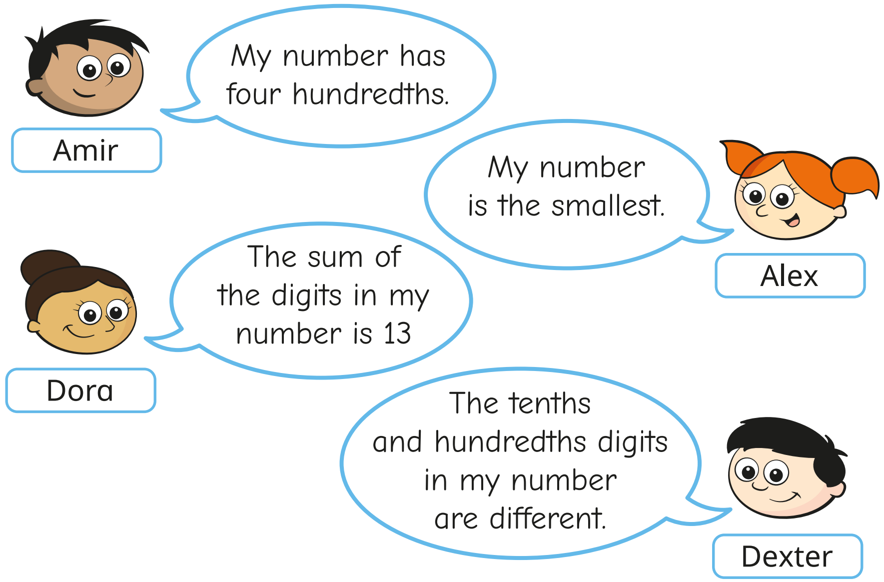
What is the value of each digit in your number?

How many ways can you partition it?

multiple possible answers

0.454 0.44 0.445 0.345

The children are each thinking of a different decimal number.



Match each number to the correct child.

Amir: 0.44 Alex: 0.345 Dora: 0.445 Dexter: 0.454

Place value – integers and decimals

Notes and guidance

In this small step, children continue to explore numbers with 3 decimal places, now extending to numbers greater than 1

As in the previous step, children use counters and place value charts to represent numbers greater than 1 with up to 3 decimal places, identify the value of the digits in a decimal number and partition decimal numbers in a range of ways. They can describe the difference between integer and decimal parts of numbers, for example recognising 3 tens and 3 tenths.

Children understand the relationship between the different place value columns, for example knowing that tenths are 10 times the size of hundredths and one-tenth the size of ones ($0.01 \times 10 = 0.1$, $1 \div 10 = 0.1$). Number lines and thousand squares are helpful representations for exploring these relationships.

Things to look out for

- Children may confuse the words "thousand" and "thousandth", "hundred" and "hundredth", and "ten" and "tenth".
- Children may use the incorrect number of placeholders, and so write the incorrect number.

Key questions

- What does a decimal number represent?
- How many tenths/hundredths/thousandths are there in 1 whole?
- How many thousandths are there in 1 hundredth?
- What digit is in the ______ column?
- What is the value of the digit ______ in the number ______?
- Which is greater, 1.897 or 3.1? How do you know?

Possible sentence stems

- There are ______ ones, ______ tenths, ______ hundredths and ______ thousandths.
  The number is ______
- There are ______ in ______
- ______ is 10/100/1,000 times the size of ______
- ______ is one-tenth/hundredth/thousandth the size of ______

National Curriculum links

- Identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places

Place value – integers and decimals

Key learning

- Use the cards to complete the sentences in as many ways as possible.

tens | ones | tenths | hundredths | thousandths

_______ are 10 times the size of _______

_______ are one-tenth the size of _______

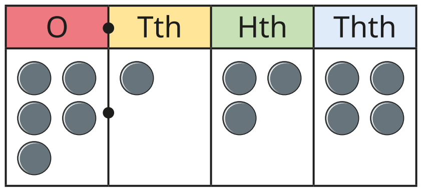
_______ are 100 times the size of _______

_______ are one-hundredth the size of _______

_______ are 1,000 times the size of _______

_______ are one-thousandth the size of _______

- Complete the sentences to describe the number.



There are _______ ones, _______ tenth, _______ hundredths and _______ thousandths.

The number is _______

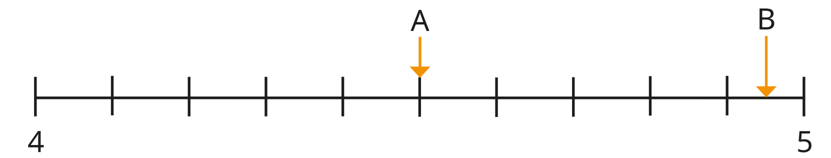
- Use a place value chart and plain counters to represent the numbers.

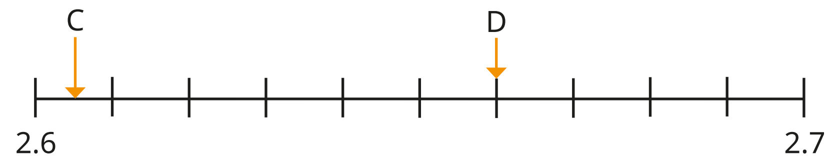
2.423 | 12.67 | 20.451 | 6.015 | 30.303

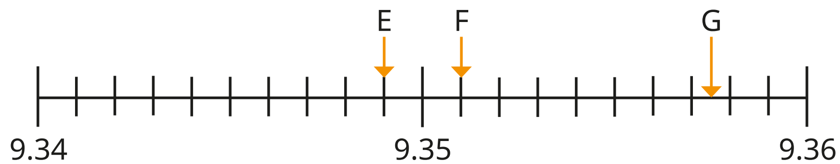
- What is the value of the 3 in each number?

1.34 | 31.4 | 11.403 | 4.13 | 103.414

- What decimal numbers are the arrows pointing to?







Place value – integers and decimals

Reasoning and problem solving

Which is the odd one out?

A 2 + 0.1 + 0.02 + 0.003

B 1 + 1.1 + 0.02 + 0.003

C 2 + 1.1 + 0.03

D 2 + 0.1 + 0.01 + 0.013

E 2 + 0.1 + 0.023

Explain your answer.

Create your own question like this for a partner.

C

C is 3.13, but all the other numbers are 2.123

O	Tth	Hth	Thth

Use five plain counters to make a number greater than 1

What is the value of each digit in your number?

How many ways can you partition it?

multiple possible answers

Is the statement always true, sometimes true or never true?

A number with 3 decimal places is greater than a number with only 1 decimal place.

Explain your answer.

sometimes true

Round decimals

Notes and guidance

In Year 5, children learnt to round numbers with up to 2 decimal places to the nearest integer and to 1 decimal place. It may be helpful to recap some of this learning before beginning this step. In this small step, children round numbers with up to 3 decimal places to the nearest integer and tenth (1 decimal place), as well as rounding to the nearest hundredth (2 decimal places) for the first time.

It is vital that children can identify the multiples of 1, 0.1 and 0.01 before and after any number with up to 3 decimal places. Children can then explore which multiple is closer, to help decide what a number should be rounded to. As with all rounding, the use of number lines can help with this process. Children recognise that when asked to round to a given degree of accuracy, they look at the place value column to the right; if the digit is 0 to 4, they round to the previous multiple and if it is 5 to 9, they round to the next multiple.

Things to look out for

- The phrase "round down" can lead children to round too low, for example rounding 6.923 down to 6.91 rather than 6.92

Key questions

- What is the next/previous integer/tenth/hundredth?
- Using the number line, which multiple of ______ is ______ closer to?
- If you are rounding to the nearest ______, which column do you need to look at to decide where to round to?
- If the digit in this column is between 0 and 4, which multiple should you round to?
- Which multiple should you round to if the digit is a 5?

Possible sentence stems

- The previous/next multiple of ______ is ______

  ______ is closer to ______ than ______

  So ______ rounded to the nearest ______ is ______

National Curriculum links

- Solve problems which require answers to be rounded to specified degrees of accuracy

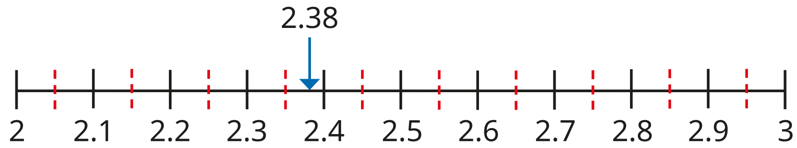
Round decimals

Key learning

- Complete the table.

Number	3.472	2.196	0.804
Previous integer	3		
Next integer	4		
Previous tenth	3.4		
Next tenth	3.5		
Previous hundredth	3.47		
Next hundredth	3.48		

- Use the number line to complete the sentences.



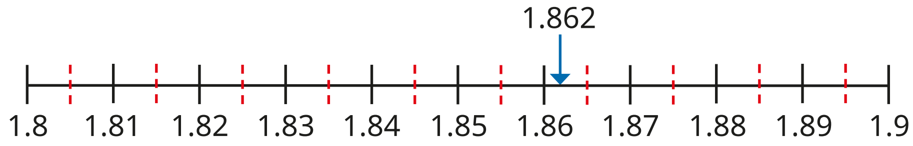
2.38 is closer to 2 than 3

2.38 rounded to the nearest integer is ______

2.38 is closer to 2.4 than 2.3

2.38 rounded to the nearest tenth is ______

- Use the number line to complete the sentences.



1.862 is closer to ______ than ______

1.862 rounded to the nearest hundredth is ______

- Complete the sentences to round 4.615 to different degrees of accuracy.
  - 4.615 is closer to ______ than ______

    4.615 rounded to the nearest hundredth is ______
  - 4.615 is closer to ______ than ______

    4.615 rounded to the nearest tenth is ______
  - 4.615 is closer to ______ than ______

    4.615 rounded to the nearest integer is ______

- Round the numbers to the nearest hundredth, tenth and integer.

2.473 | 10.185 | 7.084 | 19.987

Round decimals

Reasoning and problem solving

Here are some number cards.

4.545 4.544 5.445

5.444 4.455

Use each number once only to complete the sentences.

_______ rounded to the nearest tenth is 4.5

_______ rounded to the nearest integer is 4

_______ rounded to the nearest tenth is 5.4

_______ rounded to the nearest hundredth is 5.45

_______ rounded to the nearest hundredth is 4.54

4.545 4.455 5.444 5.445 4.544

Use the digit cards to make the statements correct.

4 5 6 7

You may use each card once only.

$\boxed{}$.803 rounded to the nearest integer is 6

5.9$\boxed{}$ rounded to the nearest tenth is 6

6.$\boxed{}$ rounded to the nearest integer is 6

$\boxed{}$.002 rounded to the nearest hundredth is 6

5.803 5.97 6.4 6.002

Add and subtract decimals

Notes and guidance

In Year 5, children added and subtracted numbers with up to 3 decimal places. In this small step, children revise the methods used for adding and subtracting numbers with different numbers of decimal places and numbers where exchanging between columns is needed.

Use place value counters in a place value chart alongside the formal written method to help children with their understanding. Begin with the smallest place value column when adding or subtracting, while at each stage asking: "Can you make an exchange?" Care must be taken when numbers have the same number of digits, but belong in different place value columns, for example 1.23 + 45.6. The use of zero placeholders can support with this. Bar models and part-whole models can be used alongside concrete resources to help children understand what calculation needs to take place.

Things to look out for

- Children may not line up digits in the correct place value columns.
- When an exchange is needed in addition, children may forget to add the exchanged number.
- Children may forget to put the decimal point in their answer.

Key questions

- How can you represent this question using place value counters?
- Do you have enough ______ to make an exchange?
- Do you need to exchange any ______?
- What are 10 tenths/10 hundredths/10 thousandths equal to?
- If there are not enough tenths/hundredths/thousandths for the subtraction, what do you need to do?

Possible sentence stems

- ______ added to ______ is equal to ______
- ______ subtract ______ is equal to ______
- ______ tenths added to ______ tenths is equal to ______ tenths.

I do/do not need to make an exchange because …

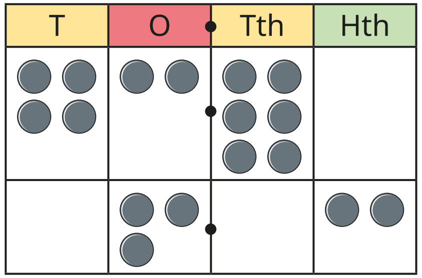
National Curriculum links

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Add and subtract decimals

Key learning

- Whitney is working out 42.6 + 3.02 using a place value chart.



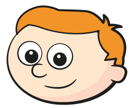
$$\begin{array}{r} 42.60 \\ +\ \ 3.02 \\ \hline 45.62 \end{array}$$

Use Whitney's method to work out the calculations.

503.6 + 25.35	56.95 – 32.8	31.67 + 1.319	249.45 – 18.3

- Ron is finding the total of 0.64 and 0.27

I will need to exchange 10 hundredths for 1 tenth.



How does Ron know this?

Use a place value chart and counters to find the total of 0.64 and 0.27

- Use a place value chart and counters to complete the calculations.

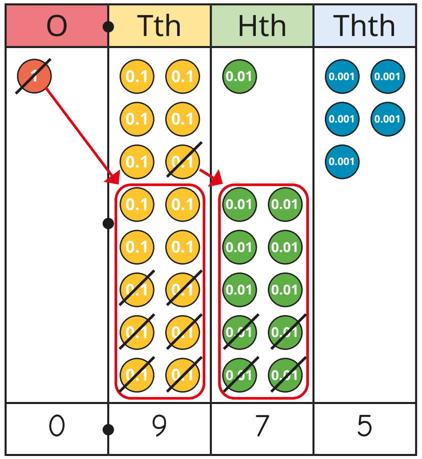
0.46 + 0.28	0.73 – 0.29	1.067 + 0.274	23.517 – 12.187

- Use place value counters to show that 1.035 + 0.18 = 1.215

- Use a place value chart to help work out the calculations.

0.468 + 1.25	5.687 + 0.97	15.027 + 9.58

- Esther uses place value counters to work out 1.615 – 0.64



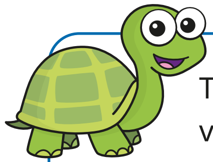
$$\begin{array}{r} {}^{0}\cancel{1}.{}^{15}\cancel{6}\,{}^{1}1\,5 \\ -\ 0.6\,4 \\ \hline 0.9\,7\,5 \end{array}$$

Use Esther's method to work out the calculations.

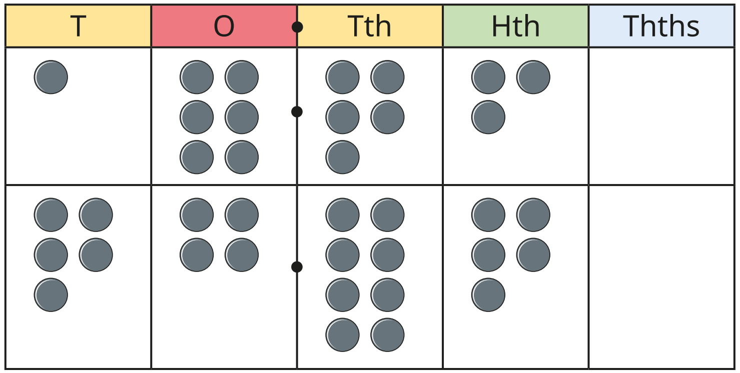
0.468 – 0.28	5.71 – 0.815	16.904 – 7.85

Add and subtract decimals

Reasoning and problem solving



Tiny has represented 16.53 + 5.485 on a place value chart.



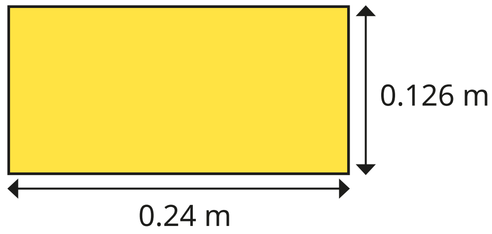
What mistake has Tiny made?

Represent the calculation correctly.

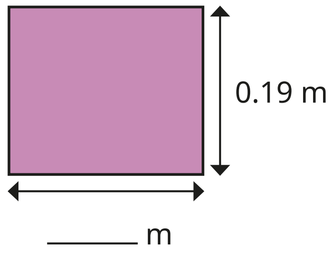
What is the correct answer?

22.015

Work out the perimeter of this shape.



This rectangle has a perimeter of 0.866 m.



Work out the missing length.

0.732 m

0.243 m

Multiply by 10, 100 and 1,000

Notes and guidance

In Year 5, children multiplied numbers with up to 2 decimal places by 10, 100 and 1,000. This small step extends to numbers with up to 3 decimal places.

Children use place value counters to represent multiplying a decimal number by 10, leading to an exchange being needed. Children see that when multiplying by 10, they exchange for a counter that goes in the place value column to the left. Children then explore how multiplying by 100 is the same as multiplying by 10 and then 10 again, so digits move two place value columns to the left. Finally, they look at multiplying by 1,000

A Gattegno chart and plain counters in a place value chart are also used to help children with their understanding.

Things to look out for

- Children may add a zero when multiplying a decimal number by 10, or two zeros when multiplying by 100, for example 5.13 × 10 = 5.130
- Children may think of the multiplication as moving the decimal point, but it is important to refer to the digits moving instead as they become, for example, 10 times greater.

Key questions

- How can you represent multiplying a decimal number with place value counters?
- What number is 10 times the size of ______?
- What number is 100 times the size of ______?
- What number is 1,000 times the size of ______?
- How can you multiply decimal numbers using a Gattegno chart?
- How can you use counters on a place value chart to multiply numbers by 10/100/1,000?

Possible sentence stems

- ______ is 10/100/1,000 times the size of ______
- ______ is one-tenth/hundredth/thousandth the size of ______
- To multiply by ______, I move the digits ______ places to the ______

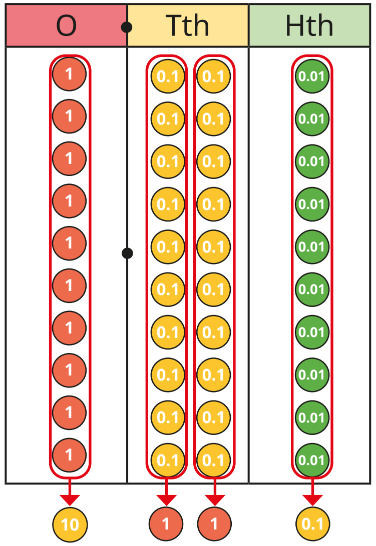
National Curriculum links

- Identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places

Multiply by 10, 100 and 1,000

Key learning

- Tommy uses place value counters to multiply 1.21 by 10



1.21 × 10 = 12.1
12.1 is 10 times the size of 1.21
1.21 is one-tenth the size of 12.1

Use Tommy's method to work out the calculations and complete the sentences for each one.

2.43 × 10 | 1.05 × 10 | 0.03 × 10 | 4.1 × 10

______ × 10 = ______

______ is 10 times the size of ______

______ is one-tenth the size of ______

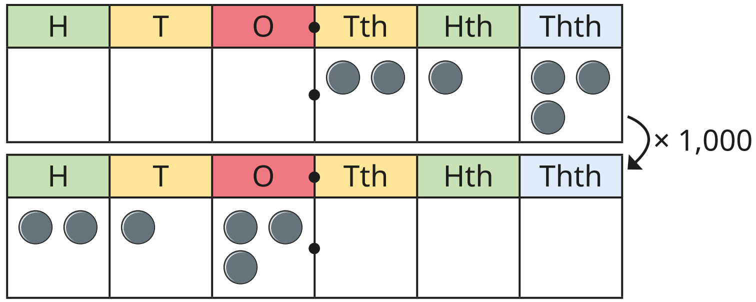
- Jack uses a Gattegno chart to work out that 0.46 × 100 = 46

10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Use a Gattegno chart to work out the calculations.

- ▶ 0.19 × 100
- ▶ 2.05 × 100
- ▶ 1.513 × 100

- Nijah multiplies 0.213 by 1,000 using a place value chart.



0.213 × 1,000 = 213
213 is 1,000 times the size of 0.2130.123 is
one-thousandth the size of 213

Use Nijah's method to work out the calculations.

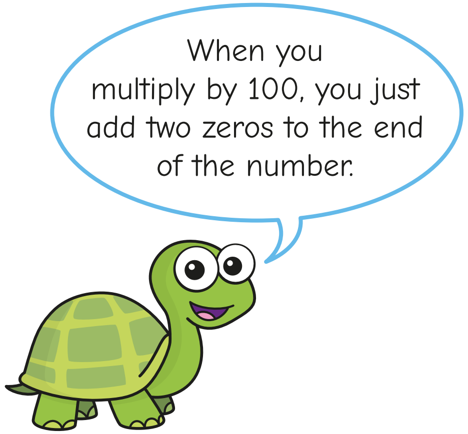
0.32 × 1,000 | 0.298 × 1,000 | 1.045 × 1,000 | 5.407 × 1,000

White Rose Maths

Multiply by 10, 100 and 1,000

Reasoning and problem solving

Tiny is multiplying numbers by 100



Give an example of a calculation where Tiny's method works.

Give an example of a calculation where Tiny's method does **not** work.

What is a better way to explain how to multiply by 100?

Talk about it with a partner.

e.g. 3 × 100

e.g. 0.3 × 100

Fill in the missing numbers.

input	→	→	→	output
0.15	× 100		× 10	
	× 10	0.09	× 10	
0.365		3.65	× 1,000	
				5,490

15, 150

0.009, 0.9

× 10, 3,650

multiple possible answers, e.g. 0.549, × 10, 5.49, × 1,000

Divide by 10, 100 and 1,000

Notes and guidance

In the previous step, children multiplied numbers with up to 3 decimal places by 10, 100 and 1,000. In this small step, they divide whole and decimal numbers by 10, 100 and 1,000. The answers will never have more than 3 decimal places.

Children use place value counters to represent a decimal number being divided by 10. As with the previous step, using language such as "10 times the size" and "one-tenth of the size" will support children in their understanding.

Children recognise that dividing a number by 10 twice is the same as dividing the number by 100. They then use a place value chart with counters (and then digits) to divide a number by 10, 100 or 1,000 by moving the counters the correct number of places to the right. A Gattegno chart used in the same way as in the previous step will also help children understand what happens to numbers as they are divided by powers of 10

Things to look out for

- Children may try to remove a zero when dividing by 10, two zeros when dividing by 100 and so on.
- Children may move the decimal point as well as the digits. Encourage them to move digits to the right as they become, for example, one-tenth of the size.

Key questions

- How can you represent dividing a decimal number with place value counters?
- What is one-tenth the size of ______?
- What is one-hundredth the size of ______?
- What is one-thousandth the size of ______?
- How can you divide decimal numbers using a Gattegno chart?
- How can you use counters on a place value chart to divide numbers by 10/100/1,000?

Possible sentence stems

- ______ is 10/100/1,000 times the size of ______
- ______ is one-tenth/hundredth/thousandth the size of ______
- To divide by ______, I move the digits ______ places to the ______

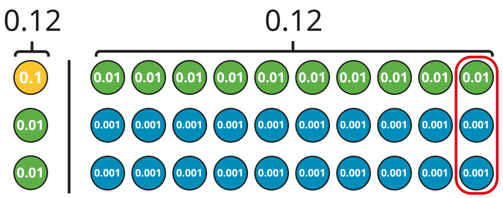
National Curriculum links

- Identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places

Divide by 10, 100 and 1,000

Key learning

- Alex divides 0.12 by 10 using place value counters.



1 tenth = 10 hundredths
1 hundredth = 10 thousandths
0.12 ÷ 10 = 0.012

Use Alex's method to work out the calculations and complete the sentences for each one.

2.43 ÷ 10	1.05 ÷ 10	0.03 ÷ 10	4.1 ÷ 10

_______ is 10 times the size of _______

_______ is one-tenth the size of _______

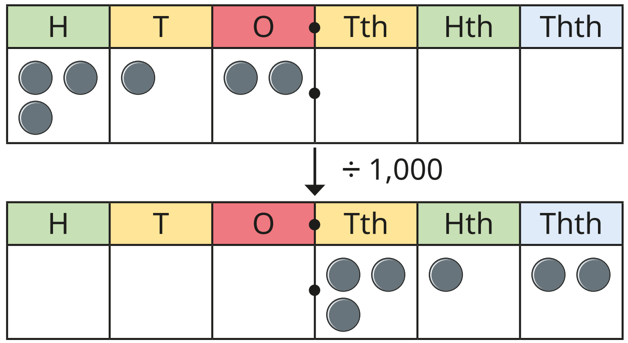
- Here are two division facts.

2.5 ÷ 10 = 0.25	0.25 ÷ 10 = 0.025

  - Explain why this means that 2.5 ÷ 100 = 0.025
  - Use this method to work out the divisions.

6.1 ÷ 100	0.8 ÷ 100	25.3 ÷ 100	7 ÷ 100

- Amir uses a place value chart to divide 312 by 1,000



312 ÷ 1,000 = 0.312
312 is 1,000 times the size of 0.312
0.312 is one-thousandth the size of 312

Use Amir's method to work out the divisions.

9 ÷ 1,000	45 ÷ 1,000	508 ÷ 1,000	2,060 ÷ 1,000

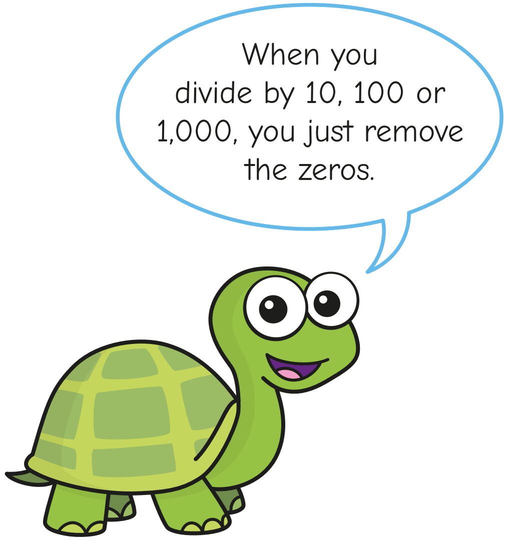
- Complete the table.

	30	3 kg			
÷ 10			0.9		
÷ 100					0.09
÷ 1,000				9	

Divide by 10, 100 and 1,000

Reasoning and problem solving

Tiny is dividing numbers by 10, 100 and 1,000



Do you agree with Tiny?

Explain your answer.

No

For example:
For 24 ÷ 10, there are no zeros to remove.

For 107 ÷ 10, you cannot just remove the zero to leave 17

Use the rules and the table to make 70 in as many ways as you can.

- Use a number from column A.
- Use an operation from column B.
- Use a number from column C.

A	B		C
7	×	÷	1
70			10
700			100
7,000			1,000

multiple possible answers, e.g.
7 × 10

Is the statement true or false?

Dividing by 1,000 is the same as dividing by 10 three times.

Explain your answer.

True

Multiply decimals by integers

Notes and guidance

In this small step, children multiply numbers with up to 2 decimal places by integers other than 10, 100 and 1,000 for the first time.

Children look at related multiplication facts using concrete resources such as place value counters, exploring relationships such as 3 × 2 = 6 and 0.3 × 2 = 0.6, and 5 × 5 = 25 and 0.5 × 5 = 2.5. They then multiply numbers with up to 2 decimal places by 1-digit integers using rows of place value counters, exchanging when needed. This is a good opportunity to explore calculations with money.

Most of the learning focuses on multiplying by a 1-digit number, but it may be appropriate to explore methods for multiplying by a 2-digit number, for example partitioning the integer and using knowledge of multiplying by 10 to support the workings:
0.4 × 14 = (0.4 × 10) + (0.4 × 4).

Things to look out for

- Children may make mistakes with exchanges where decimals are involved, for example thinking that 0.5 × 3 = 0.15
- When using related facts to multiply decimals, children may put the answer as 100 times smaller instead of 10 times smaller, for example 1.2 × 3 = 0.36

Key questions

- What is an integer?
- If you know 3 × 2 = 6, what else do you know?
- How can you show multiplying decimals by integers using counters?
- How is multiplying decimal numbers similar to/different from multiplying whole numbers?
- Do you have enough hundredths/tenths/ones to make an exchange?

Possible sentence stems

- I need to exchange 10 ______ for 1 ______
- I know that ______ × ______ = ______, so I also know that ______ × ______ = ______
- ______ multiplied by ______ is equal to ______

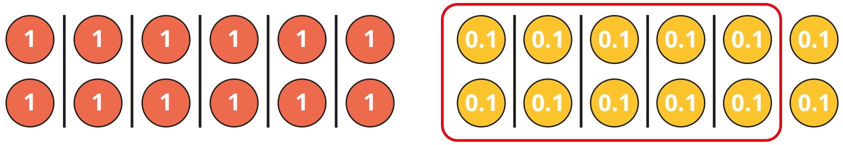
National Curriculum links

- Multiply 1-digit numbers with up to 2 decimal places by whole numbers

Multiply decimals by integers

Key learning

- Dora uses place value counters to show that 6 lots of 2 is 12, and 6 lots of 0.2 is 1.2



$6 \times 2 = 12$ $6 \times 0.2 = 1.2$

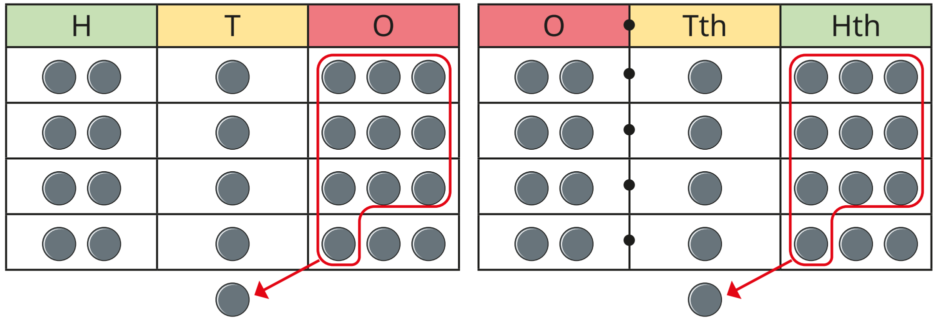
Use Dora's method to complete the calculations.

▸ 4 × 2 ▸ 5 × 5 ▸ 3 × 4 ▸ 12 × 3

4 × 0.2 0.5 × 5 3 × 0.4 1.2 × 3

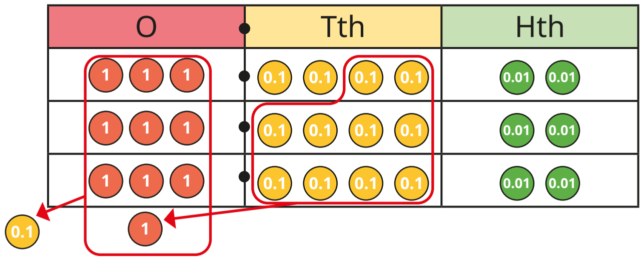
- Aisha and Filip are using counters to work out multiplications.

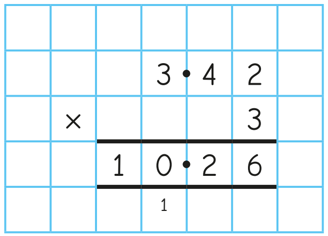
Aisha: 213 × 4 = 852 **Filip:** 2.13 × 4 = 852



What is the same and what is different about their calculations?

- Dexter uses place value counters to work out 3.42 × 3



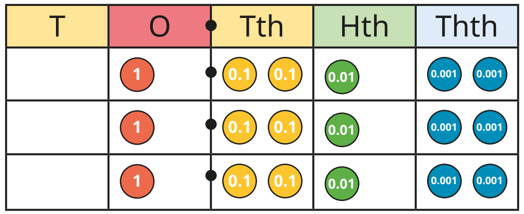


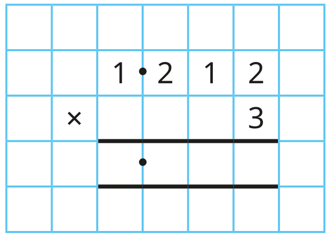
Use Dexter's method to work out the multiplications.

2.31 × 4	3.75 × 3	0.55 × 2	1.08 × 3

- Use the place value counters to multiply 1.212 by 3

Complete the calculation.



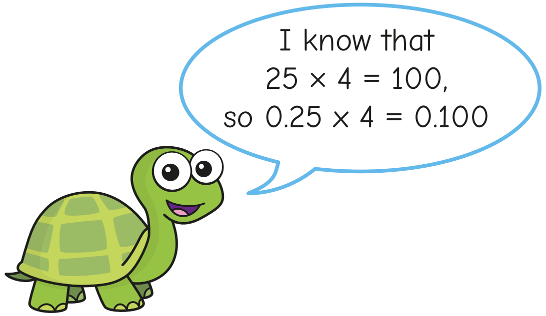


- Use place value counters and a formal written multiplication to work out the calculations.

2.121 × 4	0.613 × 5	4.056 × 3

Multiply decimals by integers

Reasoning and problem solving



Do you agree with Tiny?

Explain your answer.

No

Is the statement always true, sometimes true or never true?

When you multiply a number with 2 decimal places by an integer, the answer will have 2 decimal places.

Explain your answer.

sometimes true

Chocolate eggs can be bought individually, or in packs of 6 or 8

1 egg
52p

6 eggs
£2.85

8 eggs
£4

What is the cheapest way for Max to buy 25 chocolate eggs?

How much will he spend?

four packs of 6 plus an individual egg

£11.92

Divide decimals by integers

Notes and guidance

In this small step, children divide decimals by integers other than 10, 100 or 1,000 for the first time.

Children look at related division facts, such as 8 ÷ 2 = 4 therefore 0.8 ÷ 2 = 0.4 and 0.08 ÷ 2 = 0.04. Explore the pattern that as the number being divided becomes 10 or 100 times smaller, the answer becomes 10 or 100 times smaller, modelling this using place value counters in a place value chart.

Children explore a range of division facts using times-table knowledge, for example 144 ÷ 12 = 12, so 1.44 ÷ 12 = 0.12. Using place value counters, children put counters into groups, starting with the greatest place value column. They start with division where no exchanges are needed before moving on to calculations needing exchanges. They use the formal written method for division alongside the place value charts.

Things to look out for

- When using related facts, children may make the number being divided one-hundredth the size, but only make the answer one-tenth the size, for example 8 ÷ 2 = 4, so 0.08 ÷ 2 = 0.4
- When using the formal written method for division, children may forget to add the decimal point.

Key questions

- If you know that ______ ÷ ______ = ______, what else do you know?
- If you make the number being divided one-tenth the size, what must you do to the answer?
- How can you show this division using place value counters?
- How many groups of ______ can you make with ______?
- What happens to tenths or hundredths that you cannot group?

Possible sentence stems

- I know that ______ ÷ ______ is ______, so I also know that ______ ÷ ______ is ______
- If ______ ones divided by ______ is equal to ______, then ______ tenths/hundredths divided by ______ is equal to ______

National Curriculum links

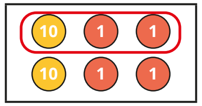
- Use written division methods in cases where the answer has up to 2 decimal places

Divide decimals by integers

Key learning

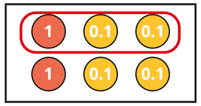
- Dani, Mo and Kim use place value counters to work out divisions.

Dani



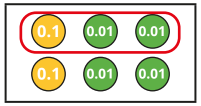
24 ÷ 2 = 12

Mo



2.4 ÷ 2 = 1.2

Kim



0.24 ÷ 2 = 0.12

What is the same about their divisions?

What is different about their divisions?

What do you notice?

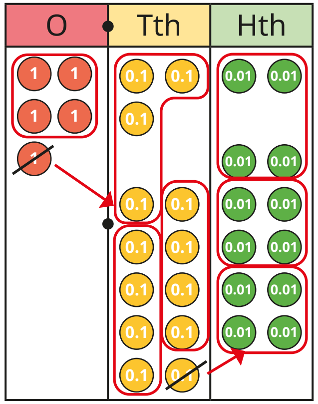
- Use place value counters to work out the divisions.
  - 4 ÷ 2 and 0.4 ÷ 2
  - 9 ÷ 3 and 0.09 ÷ 3
  - 36 ÷ 6 and 3.6 ÷ 6
  - 15 ÷ 3 and 0.15 ÷ 3

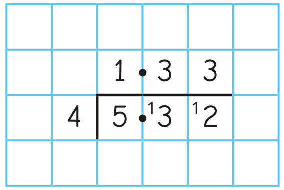
- Use counters and a place value chart to work out the divisions.

8.46 ÷ 2   0.84 ÷ 2   9.36 ÷ 3   9.03 ÷ 3

- Scott uses place value counters in a place value chart to work out 5.32 ÷ 4

He writes his calculation using the formal written method.





Use place value counters alongside the formal written method to work out the divisions.

3.12 ÷ 2   7.32 ÷ 3   6.05 ÷ 5

- Max has £7.48

He shares this money equally between him and 5 friends.

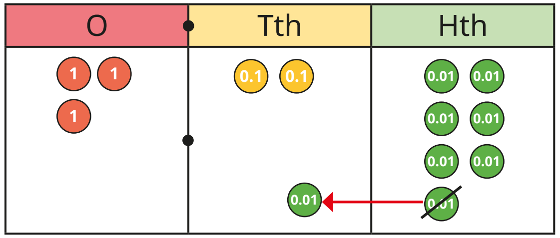
He puts the money left over in a pot.

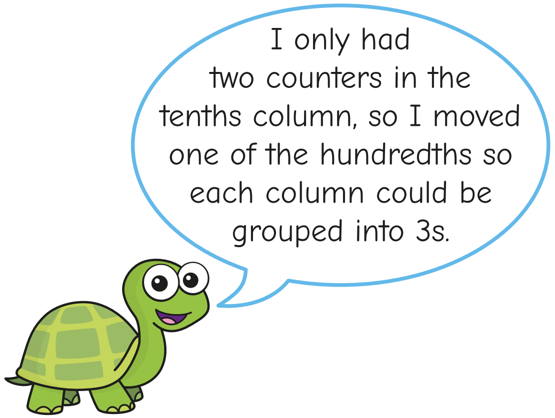
How much money does he put in the pot?

Divide decimals by integers

Reasoning and problem solving

Tiny uses place value counters to work out 3.27 ÷ 3





Explain why Tiny is incorrect.

What is the correct answer?

1.09

C is $\frac{1}{4}$ of A

B = C + 2

Use this information to complete the division.

		0 •	B	B	
	A	C •	^{C}B	^{C}2	

Compare methods with a partner.

How did you work it out?

Create your own question like this for someone else to solve.

A = 4

B = 3

C = 1

White Rose Maths

Multiply and divide decimals in context

Notes and guidance

This small step takes the skills explored in the previous two steps and applies them in a variety of contexts and problems.

Children recap the formal written methods for both multiplication and division alongside place value counters. They can use the same method with coins, with £1 coins replacing the ones, 10p coins replacing the tenths and 1p coins replacing the hundredths. Children then use these skills in a variety of contexts to solve problems.

Encourage children to use bar models to help them to identify what operation is needed and in what order steps should be taken.

It may be useful to recap conversions of units of measure from earlier in the year before beginning this step.

Things to look out for

- Children may be unsure which operation is needed to solve a problem.
- When solving questions in context, children may forget the units of measure.
- If a unit conversion is needed, for example kilograms to grams, children may multiply or divide by the incorrect amount.

Key questions

- How can you tell what operation you need to perform to answer this question?
- How can you represent this question using place value counters?
- What do you need to work out?
- How can you draw a bar model to represent this problem?
- Do you need to convert any units of measure to answer this question?

Possible sentence stems

- ______ multiplied by ______ is ______
- ______ divided by ______ is ______

National Curriculum links

- Multiply 1-digit numbers with up to 2 decimal places by whole numbers
- Use written division methods in cases where the answer has up to 2 decimal places
- Solve problems involving addition, subtraction, multiplication and division

Multiply and divide decimals in context

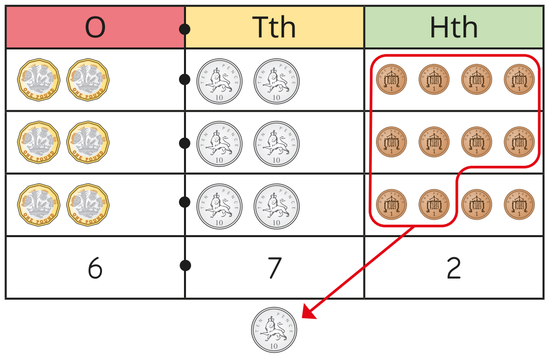
Key learning

- The table shows the prices of items in a shop.

Item	Cost
Magazine	£2.24
Book	£5.25
CD	£3.49
DVD	£4.75

Esther wants to buy three magazines.

She uses coins in a place value chart alongside the formal written method to work out the total cost.



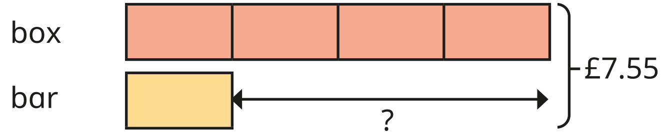
		2 •	2	4	
	×			3	
		6 •	7	2	
		1			

Use Esther's method to work out the costs of these items.

4 books | 3 CDs | 5 DVDs and 6 books

- A box of chocolates costs 4 times as much as a chocolate bar.

Together they cost £7.55



How much more does the box of chocolates cost than the chocolate bar?

- Modelling clay is sold in two different shops.
  - Shop A sells 4 pots of clay for £7.68
  - Shop B sells 3 pots of clay for £5.79

Which shop has the better deal?

Explain your answer.

- Huan has 9.6 litres of juice.

He fills 8 identical jugs with the juice.

How many millilitres of juice does each jug hold?

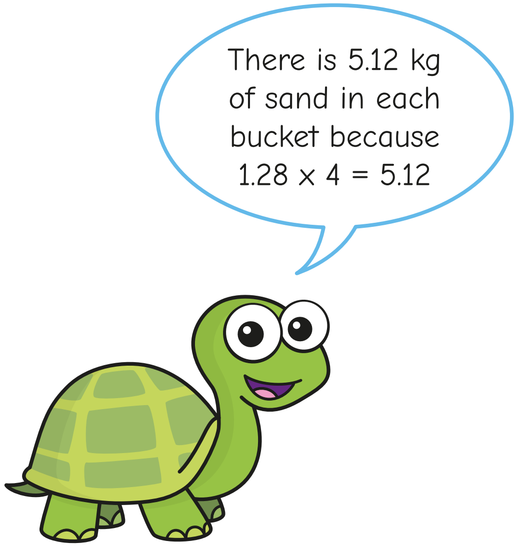
- A square has a perimeter of 0.824 m.

How long is each side?

Multiply and divide decimals in context

Reasoning and problem solving

1.28 kg of sand is shared equally between 4 buckets.



Explain the mistake that Tiny has made.

What is the mass of sand in each bucket?

0.32 kg

Annie has some money.

- She gives $\frac{2}{3}$ of her money to charity.
- She then buys three footballs costing £6.45 each.
- Her mum gives her and her two sisters £9.75 to share equally between them.

How much money did Annie have to start with?

£79.80

Spring Block 1

Ratio

Small steps

Step 1	Add or multiply?
Step 2	Use ratio language
Step 3	Introduction to the ratio symbol
Step 4	Ratio and fractions
Step 5	Scale drawing
Step 6	Use scale factors
Step 7	Similar shapes
Step 8	Ratio problems

Small steps

Step 9	Proportion problems
Step 10	Recipes

Add or multiply?

Notes and guidance

In this small step, children explore the fact that the relationship between two numbers can be expressed additively or multiplicatively. For example, the relationship between 3 and 9 can be expressed as an addition (3 + 6 = 9) or a multiplication (3 × 3 = 9). Children use this understanding to complete sequences of numbers, deciding whether each relationship is additive or multiplicative.

Children also explore the inverse relationships related to each of these, for example 9 – 6 = 3 and 9 ÷ 3 = 3. Using language such as "3 times the size" and "a third of the size" will support their understanding of multiplicative relationships.

Children will explore these relationships using double number lines and should be encouraged to explore all of the additive and multiplicative links that can be seen.

Things to look out for

- Children may see just additive relationships and not notice the multiplicative relationships.
- Children may not start double number lines from zero.
- When using double number lines, children may focus on the horizontal relationships and not notice the vertical relationships.

Key questions

- How can you describe the relationship between these two numbers using addition/multiplication?
- What is the inverse of addition/multiplication?
- What addition/subtraction/multiplication/division calculations can be written from this information?
- Is the relationship in the sequence additive or multiplicative?
- How do the relationships on the upper number line relate to those on the lower number line?

Possible sentence stems

- ______ × ______ = ______ and ______ + ______ = ______
- ______ is ______ times the size of ______
- ______ is $\frac{\square}{\square}$ the size of ______

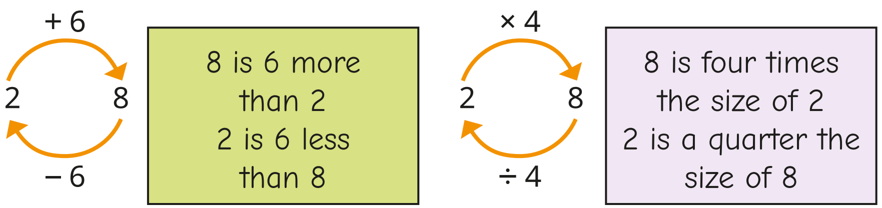
National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

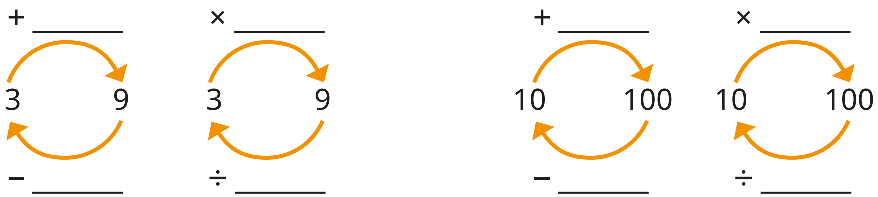
Add or multiply?

Key learning

- The relationship between 2 and 8 can be described as additive or multiplicative.



  Complete the models to show the additive and multiplicative relationships.



  Describe the relationships to a partner.

- A sequence starts 3, 6 …
  - Explain why the next number could be 9
  - Explain why the next number could be 12
  - What could the next number be in these sequences?

5, 10 …	7, 21 …	100, 50 …

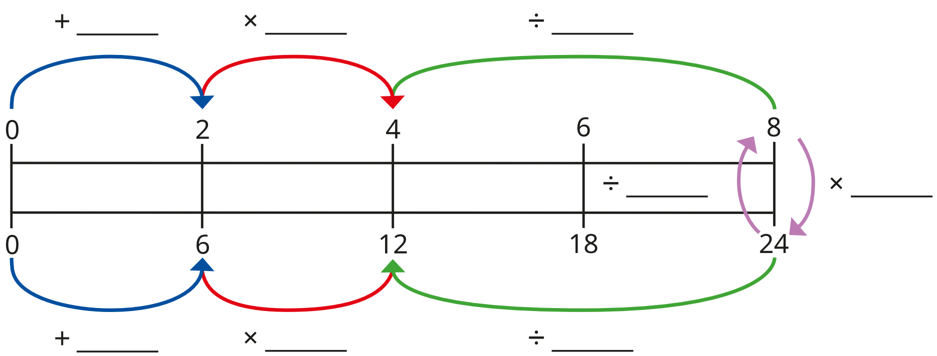
  Find two answers for each.

- Complete the sequences.
  - 4, 8, ______, 32, ______, ______
  - ______, 14, 21, 28, ______, ______
  - 1, ______, ______, 27, 81, ______

  Are the relationships additive or multiplicative?
  Could they be both?

- The double number line shows the relationship between two sets of numbers.

  Fill in the missing values to describe the relationships.



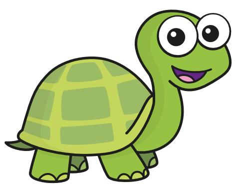
  What other additive and multiplicative relationships can you see on the double number line?

Add or multiply?

Reasoning and problem solving

6	12		
2	4	8	
4	12	20	

Each of these sequences can be completed using either addition or multiplication.



Do you agree with Tiny?

Explain your answer.

No

Here are the different options in a pizza shop.

Base	Topping
Thin	Cheese and tomato
Deep pan	Vegetarian feast
	Chicken
	Meat feast

Use both additive and multiplicative reasoning to explain why there are 8 possible combinations of base and topping.

The restaurant introduces a new topping of tuna and sweetcorn.

How many combinations are there now?

How many combinations would there be with 4 base options and 17 topping options?

Did you use additive or multiplicative relationships to work out each answer?

10 | 68

Use ratio language

Notes and guidance

In this small step, children are introduced to the idea of ratio representing a multiplicative relationship between two amounts.

Children see how one value is related to another by making simple comparisons, such as: "For every 2 blue counters, there are 3 red counters." A double number line can be used to show such relationships, building up to recognise that this example is equivalent to 4 blue, 6 red or 20 blue, 30 red and so on. At this point, relationships will only be expressed in words and the ratio symbol will be introduced in the next step.

Children move on to expressing relationships more simply. For example, if there are 10 red and 15 blue counters, these can be physically rearranged so that "For every 2 red counters, there are 3 blue counters." Children can link this to dividing by a common factor, 5, and relate this to their understanding of simplifying fractions.

Things to look out for

- Children may use additive rather than multiplicative relationships to make comparisons, for example "There is one more blue than red."

Key questions

- How can you give the relationship between the number of ______ and the number of ______?
- For every ______, how many ______ are there?
- How can you rearrange the counters to make the ratio simpler?
- What number is a common factor of ______ and ______? How can you use this to make the ratio simpler?
- How many ______ would there be if there were ______?

Possible sentence stems

- For every ______, there are ______
- If there were ______, there would be ______
- A common factor of ______ and ______ is ______

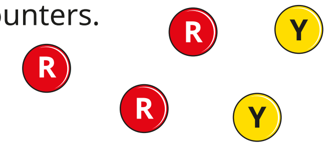
National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Use ratio language

Key learning

- Complete the sentences to describe the counters.

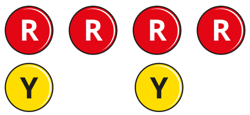


  There are ______ red counters and ______ yellow counters.

  For every ______ red counters, there are ______ yellow counters.

  For every ______ yellow counters, there are ______ red counters.

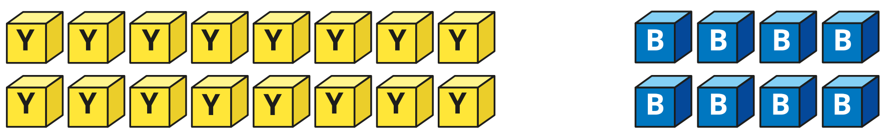
- Complete the sentence to describe the counters.



  For every ______ red counters, there is ______ yellow counter.

  Can you complete it a different way?

- Complete the sentences to describe the cubes.

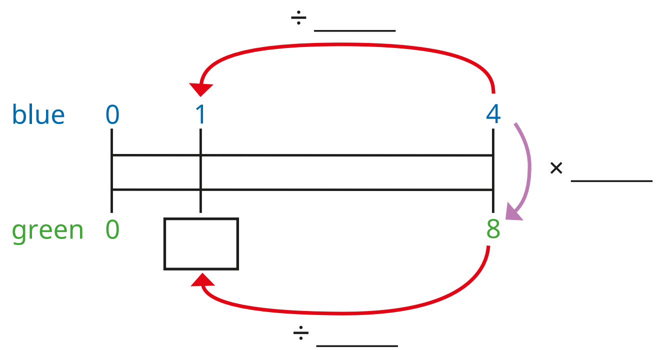


  For every 16 yellow cubes, there are ______ blue cubes.

  For every 8 yellow cubes, there are ______ blue cubes.

  For every 1 blue cube, there are ______ yellow cubes.

- Amir is using a double number line to find equivalent ratios.



  ▶ Use Amir's number line to help you complete the sentence.

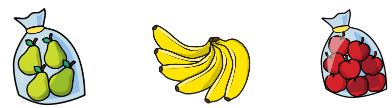
  For every 1 blue counter, there are ______ green counters.

  ▶ Use a double number line to complete the sentences.

  For every 4 green counters, there are ______ blue counters.

  For every ______ blue counters, there are 16 green counters.

- Complete the sentences to describe the fruit.



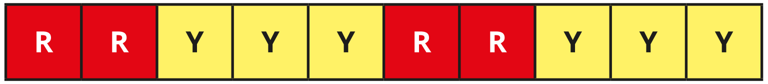
  For every ______ pears, there are ______ bananas.

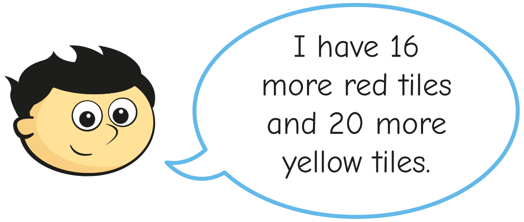
  For every ______ pears, there are ______ apples.

Use ratio language

Reasoning and problem solving

Jack puts red and yellow tiles in this pattern.





Can Jack continue this pattern without there being any tiles left over?

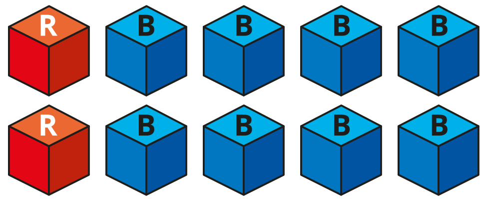
Explain your answer.

No

There are 2 red tiles for every 3 yellow tiles.

16 red tiles will need 24 yellow tiles.

Decide if each statement is true or false.



For every red cube, there are 8 blue cubes.

For every 4 blue cubes, there is 1 red cube.

For every 3 red cubes, there would be 12 blue cubes.

For every 16 cubes, 4 would be red and 12 would be blue.

Give reasons for your answers.

False

True

True

False

Introduction to the ratio symbol

Notes and guidance

In this small step, children continue to explore the multiplicative relationship between values, now seeing it written using the ratio symbol, a colon.

Explain that the wording, "For every ______, there are ______" can be written as ______ : ______. Show children that the order in which the notation is used is important. For example, for every 2 red cubes there are 3 blue cubes, so red to blue is 2 : 3. For every 3 blue cubes, there are 2 red cubes, so blue to red is 3 : 2. Ensure that children know, and convey in their answers, which number refers to which value.

Children build on the ideas of the previous step to understand that the same ratio can be written in different forms, for example 4 : 6 can be written as 2 : 3. This step is a good opportunity to use contexts such as measure, looking at the ratios of the masses of ingredients in recipes.

Things to look out for

- Children may not understand the meaning of the ratio symbol, and may confuse it with a decimal point.
- When simplifying a ratio, children may try to use additive rather than multiplicative relationships.

Key questions

- If there are 3 blue counters and 5 red counters, how can you describe the relationship between these numbers?
- What does the : symbol mean in the context of ratio?
- What does 2 : 3 mean?
- How can you compare the relationship between three quantities?
- Are the ratios 2 : 3 and 3 : 2 the same?
- How else can you write the ratio 2 : 4?

Possible sentence stems

- For every ______, there are ______, which can be written as ______ : ______
- The ratio of ______ to ______ is ______ : ______
- In the ratio ______ : ______, the first number represents ______ and the second number represents ______

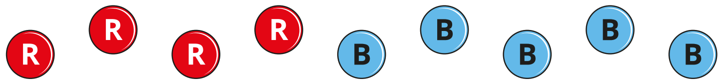
National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Introduction to the ratio symbol

Key learning

- Complete the sentences.



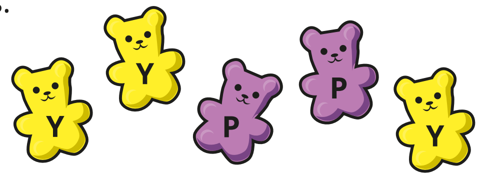
For every _______ red counters, there are _______ blue counters.

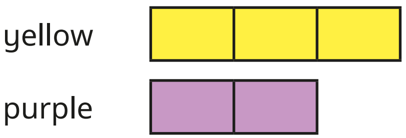
The ratio of red counters to blue counters is _______ : _______

For every _______ blue counters, there are _______ red counters.

The ratio of blue counters to red counters is _______ : _______

- Aisha draws a bar model to show the ratio of yellow to purple gummy bears.



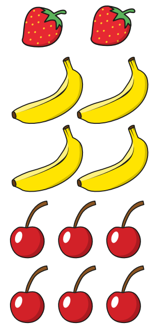


Complete the sentences.

The ratio of yellow gummy bears to purple gummy bears is _______ : _______

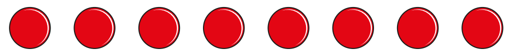
The ratio of purple gummy bears to yellow gummy bears is _______ : _______

- Write the ratio of:
  - bananas to strawberries
  - cherries to strawberries
  - strawberries to bananas to cherries
  - cherries to strawberries to bananas

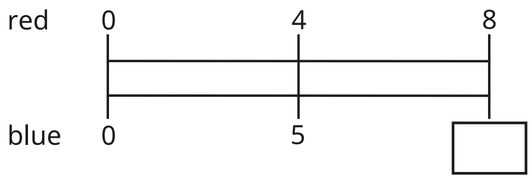


Draw a bar model to represent each ratio.

- Here are 8 red counters.



How many blue counters does he need so that the ratio of red to blue is 4 : 5?



How does the double number line help to work it out?

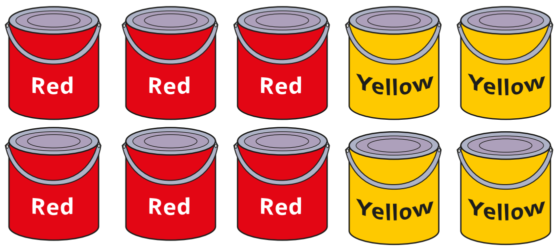
- Max has blue and red counters in the ratio 3 : 5

He has 15 blue counters.

How many red counters does he have?

Introduction to the ratio symbol

Reasoning and problem solving



Decide if each statement is true or false.

There are 2 yellow tins for every 3 red tins.

There are 2 red tins for every 3 yellow tins.

The ratio of red tins to yellow tins is 2 : 3

The ratio of yellow tins to red tins is 2 : 3

Explain your answers.

True

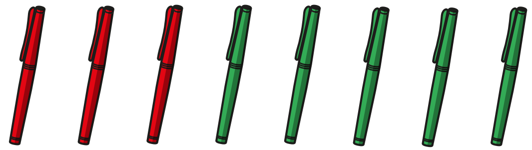
False

False

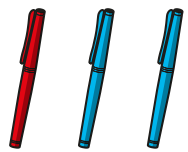
True

In a box, there are some red, blue and green pens.

The ratio of red pens to green pens is 3 : 5



For every 1 red pen, there are 2 blue pens.



There are 6 red pens in the box.

How many green pens are there?

How many blue pens are there?

Write the ratio of red pens to blue pens to green pens.

10

12

3 : 6 : 5

6 : 12 : 10

3 : 6 : 5

Ratio and fractions

Notes and guidance

In this small step, children explore the differences and similarities between ratios and fractions.

Children may have already noticed that simplifying ratios is similar to simplifying fractions and that both involve dividing by common factors. A possible misconception is thinking, for example, that the ratio 1 : 2 is the same as $\frac{1}{2}$. Exploring links between ratios and fractions using representations such as counters and bar models can help to overcome this. The key point is that a ratio compares one item with another, whereas fractions compare each part with the whole.

Children then explore ratio when given a fraction as a starting point. For example, they are told that $\frac{1}{4}$ of a group of objects is blue, and they need to find the ratio of blue to not blue. Initially, they may think the ratio is 1 : 4, but concrete resources and diagrams can support them to see it is 1 : 3

Things to look out for

- Children may not consider the whole when linking ratios and fractions. For example, they may think the 2 in 2 : 3 is $\frac{2}{3}$ rather than $\frac{2}{5}$

Key questions

- What is the ratio of one part to another?
- How many parts are there altogether?
- What fraction of the whole is the first/second/third part?
- How are fractions and ratios similar? How are they different?
- What fraction does the ratio 1 : 2 mean? Is this the same as $\frac{1}{2}$ or is it different?
- How can you represent the ratio/fraction as a bar model?

Possible sentence stems

- The ratio of ______ to ______ is ______ : ______

  There are ______ parts altogether.

  The fraction that is ______ is ______

National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Ratio and fractions

Key learning

- The ratio of red counters to blue counters in a box is 1 : 2

  R B B

  - What fraction of the counters are blue?
  - What fraction of the counters are red?
  - What is the same about the ratio and the fractions? What is different?

- This bar model represents $\frac{2}{5}$

  This bar model represents 2 : 5

  What is the same and what is different about the bar models?

- Use the diagram to complete the sentences.

  B B G G G

  The ratio of blue counters to green counters is 2 : _____

  The fraction of counters that are blue is $\frac{2}{\square}$

- One third of the chocolates in a box are mint flavoured.

  The rest are strawberry.

  Use diagrams to show that the ratio of mint to strawberry chocolates is 1 : 2

- The bar model shows the ratio 2 : 3 : 4

P	P	Y	Y	Y	B	B	B	B

  - What fraction of the bar is pink?
  - What fraction of the bar is yellow?
  - What fraction of the bar is blue?

- Esther gets $\frac{2}{5}$ of a packet of 30 sweets.

  Huan shares 70 sweets with his friend in the ratio 2 : 5

  How many more sweets does Huan get than Esther?

- Brett opens a box of buttons and counts the different colours.

  - $\frac{1}{2}$ of them are red.
  - $\frac{1}{3}$ them are green.
  - The rest are yellow.

  What is the ratio of red : green : yellow buttons in the box?

Ratio and fractions

Reasoning and problem solving

There are some red and green cubes in a bag.

$\frac{2}{7}$ of the cubes are red.

Are the statements true or false?

For every 2 red cubes, there are 7 green cubes.

For every 2 red cubes, there are 5 green cubes.

For every 5 green cubes, there are 2 red cubes.

For every 5 green cubes, there are 7 red cubes.

Explain your answers.

False

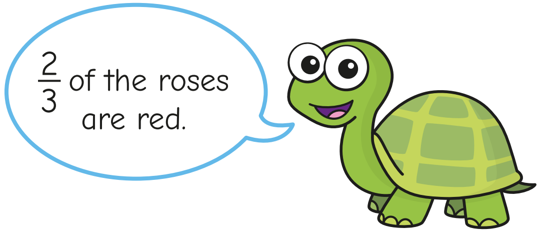
True

True

False

Mrs Fisher plants flowers in a flower bed.

For every 2 red roses, she plants 3 white roses.



Is Tiny correct?

Explain your answer.

No

Dani makes 240 ml of squash using cordial and water in the ratio 1 : 3

She adds more water to the cup so there is now 300 ml of squash.

What fraction of the drink is cordial?

$\frac{1}{5}$

Scale drawing

Notes and guidance

In this small step, children apply their understanding of ratio and multiplicative relationships through scale diagrams. Before children begin to draw, it is important to spend time exploring what scale diagrams are by getting them to decide by eye if diagrams are accurately scaled or if the proportion of the dimensions has been changed.

Children become familiar with the language of "Each square represents …" to explain the relationship between the original image and its scale drawing.

Encourage children to explore different ways of calculating scaled lengths using multiplicative relationships between numbers. For example, if 3 cm represents 9 cm, then to find what 6 cm represents they can either multiply 9 cm by 2 or multiply 6 cm by 3 to find the result, 18 cm.

Once children are confident with this and are able to draw squares and rectangles, they may move on to drawing more complex rectilinear shapes.

Things to look out for

- Children may identify the correct scale of enlargement but still become confused by whether they need to multiply or divide.

Key questions

- How do you know if a diagram is drawn to scale?
- Why might you need to draw a scale diagram?
- If 1 square represents 5 cm, what do ______ squares represent? How do you know?
- If 1 square represents 5 cm, how many squares represent ______ cm? How do you know?
- Is there more than one way of finding the missing value?
- How is a scale like a ratio?

Possible sentence stems

- ______ squares represents ______, so each square represents ______
- Each square represents ______, so ______ squares represent ______ × ______ = ______
- Each square represents ______, so ______ is represented by ______ ÷ ______ = ______ squares.

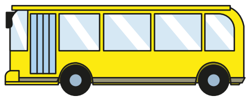
National Curriculum links

- Solve problems involving similar shapes where the scale factor is known or can be found

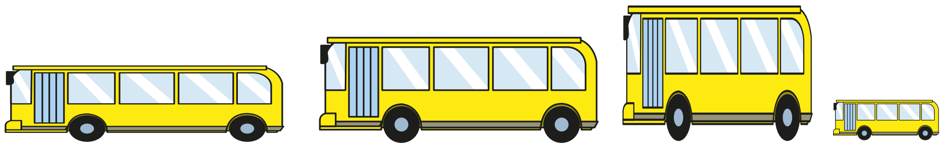
Scale drawing

Key learning

- Here is a picture of a bus.

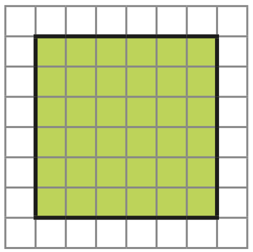


Which two pictures are scale drawings of the original?



- A square has side lengths of 12 cm.

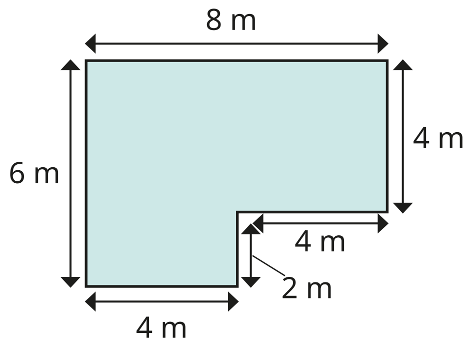
Scott has drawn a scale diagram of the shape in which the side length of each square in the grid represents 2 cm.



Use squared paper to draw other scale diagrams using the side length of each square to represent:

- 3 cm
- 4 cm
- 6 cm
- 12 cm

- This is a plan of a classroom.



Using squared paper, draw a scale diagram of the classroom if each square on the grid represents 2 m.

- A football pitch measures 48 m by 72 m.

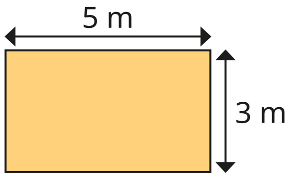
Using squared paper, draw a scale diagram of the football pitch if each square on the grid represents 8 m.

- On a scale diagram, 4 cm represents 1 m.
  - What does 8 cm represent?
  - What does 40 cm represent?
  - What does 2 cm represent?
  - What does 1 cm represent?
  - What length in centimetres would represent 3 m?

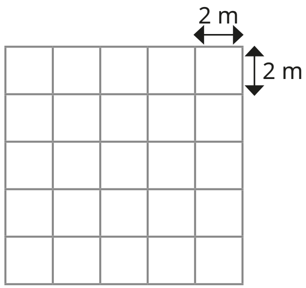
Scale drawing

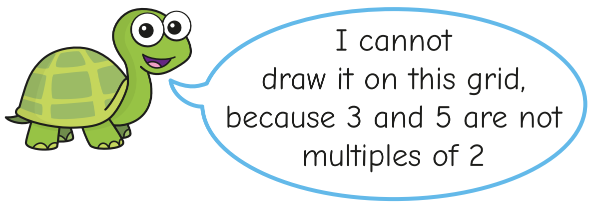
Reasoning and problem solving

Tiny wants to draw a scale diagram of this rectangle.



Each square on the grid represents 2 m.



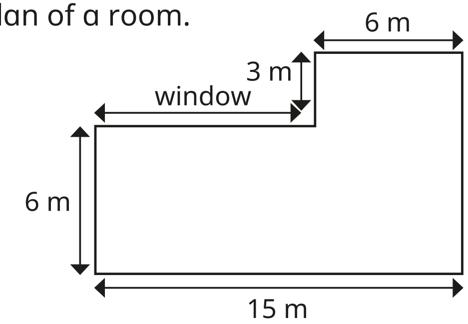


Do you agree with Tiny?

Explain your answer.

No

Here is a plan of a room.



Draw a scale diagram of the room where each square represents 3 m.

What is the actual length of the window?

What is the area, in squares, of the room in the scale diagram?

What is the actual area of the room?

Explain the connection between your answers.

9 m

12 squares

108 m^2

Use scale factors

Notes and guidance

In this small step, children build on the previous step to enlarge shapes and describe enlargements.

Children need to know that one shape is an enlargement of another if all the matching sides are in the same ratio. They can use familiar language such as "3 times as big" before being introduced to the language of scale factors, for example "enlarged by a scale factor of 3". They can then draw the result of an enlargement by a given scale factor. Children also identify the scale factor of an enlargement when presented with both images. Once confident with this, they can explore using inverse operations to find the dimensions of the original shape given the size of the enlargement.

Things to look out for

- Children may not use the scale factor with all the dimensions of the shape.
- Children may use inaccurate measuring when working with shapes with diagonal lines rather than considering the vertical and horizontal distances.

Key questions

- What does "scale factor" mean?
- How do you draw an enlargement of a shape?
- How can you work out the scale factor of enlargement between two shapes?
- If a shape has been enlarged by a scale factor of ______, how can you find the dimensions of the original shape?
- Do you need to multiply or divide to find the missing length? How do you know?

Possible sentence stems

- ______ × ______ = ______
- The shape is ______ times as big, so the scale factor of the enlargement is ______
- If a shape has been enlarged by a scale factor of ______, I need to ______ by ______ to find the original dimensions.

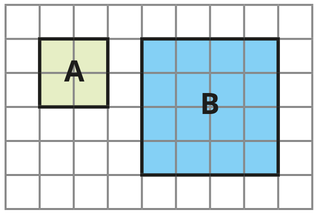
National Curriculum links

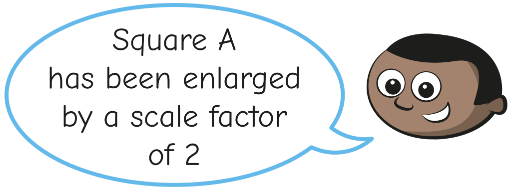
- Solve problems involving similar shapes where the scale factor is known or can be found

Use scale factors

Key learning

- Mo draws a square twice as big as square A and labels it B.

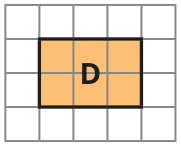




  - Draw a square that is 3 times as big as square A.

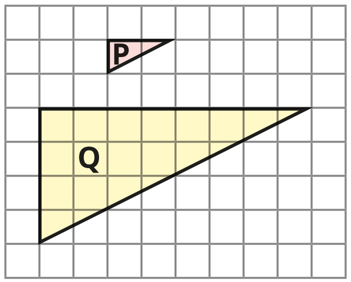
    Label it C.
  - What is the scale factor of enlargement from A to C?

- Use squared paper to complete the enlargements.

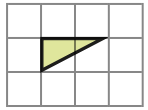


  - Enlarge rectangle D by a scale factor of 2 and label it E.
  - Enlarge rectangle D by a scale factor of 4 and label it F.

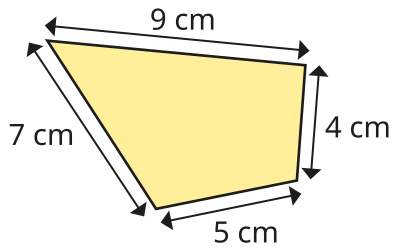
- What is the scale factor of enlargement from P to Q?



- On squared paper, enlarge the triangle by a scale factor of 3



- Here is a quadrilateral.

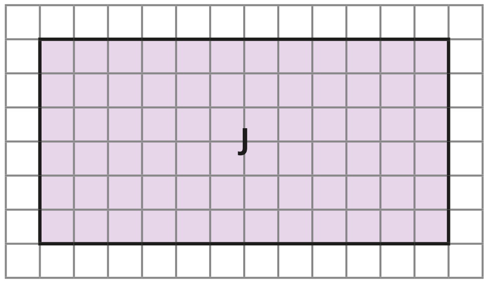


The shape is enlarged by a scale factor of 7

What are the lengths of the sides of the enlarged shape?

- A shape is enlarged by a scale factor of 3

  Shape J is the result of the enlargement.

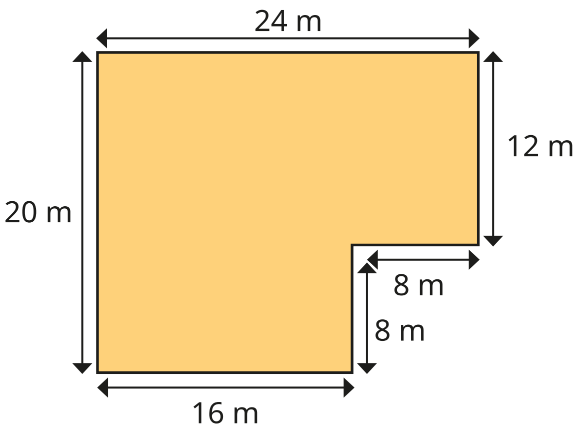


Draw the original shape.

Use scale factors

Reasoning and problem solving

The shape is the result of an enlargement by a scale factor of 4



What is the perimeter of the enlarged shape?

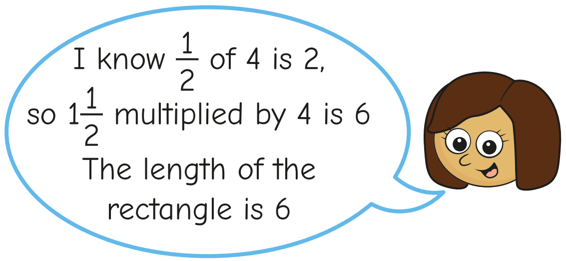
What is the perimeter of the original shape?

What do you notice?

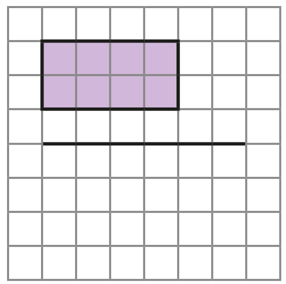
88 m

22 m

Kim is enlarging the shape by a scale factor of $1\frac{1}{2}$



Complete the enlargement.



On squared paper, enlarge the shape by a scale factor of $2\frac{1}{2}$

On squared paper, enlarge the shape by a scale factor of $1\frac{1}{4}$

side lengths 6 and 3

side lengths 10 and 5

side lengths 5 and $2\frac{1}{2}$

Similar shapes

Notes and guidance

In this small step, children build on the previous step to explore similar shapes. Similar shapes are defined as shapes where corresponding sides are in the same proportion and the corresponding angles are equal, so if one shape is an enlargement of the other, the two shapes are similar. When testing for similarity, encourage children to work systematically around a shape to ensure that all sides have been enlarged by the same scale factor.

Children can explore the relationship between corresponding angles in the shapes, practising protractor skills learnt in Year 5. Finally, children should apply this understanding to explore similar shapes that are in different orientations, identifying corresponding sides and angles to decide if the shapes are similar.

Things to look out for

- If shapes are in different orientations, children may struggle to identify corresponding sides or just believe the shapes cannot be similar because they do not look the same.
- It is important that children work systematically to ensure all corresponding sides are in the same proportion, rather than just one or two.

Key questions

- What do you think "similar" means?
- What is the scale factor of the enlargement?
- Have all the sides been enlarged by the same amount?
- What are corresponding sides? Can you identify the corresponding sides in these two shapes?
- What do you notice about corresponding angles in similar shapes?
- Does it matter that the shapes are in a different orientation?

Possible sentence stems

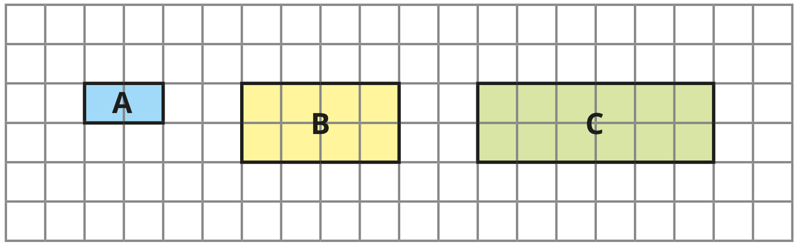
- Each side of the shape is ______ times the size, so the shape has been enlarged by a scale factor of ______. Therefore, the shapes are ______
- I know that the shapes are similar, because the corresponding sides have been enlarged by the same ______, and the corresponding angles are ______

National Curriculum links

- Solve problems involving similar shapes where the scale factor is known or can be found

Similar shapes

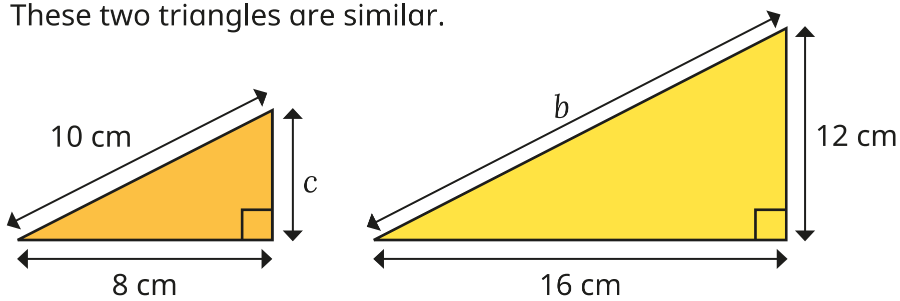
Key learning

- 

  - Explain why shapes A and B are similar.
  - Explain why shapes A and C are **not** similar.
  - Draw another shape that is similar to A.

  Compare answers with a partner.

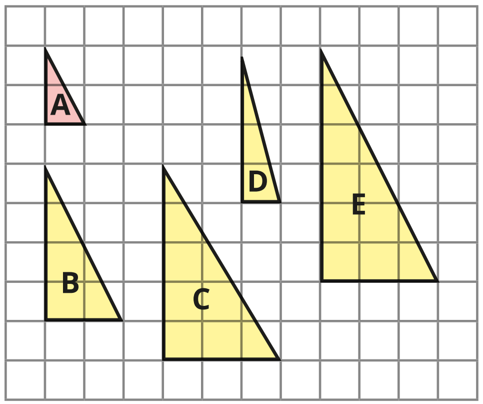
- These two triangles are similar.



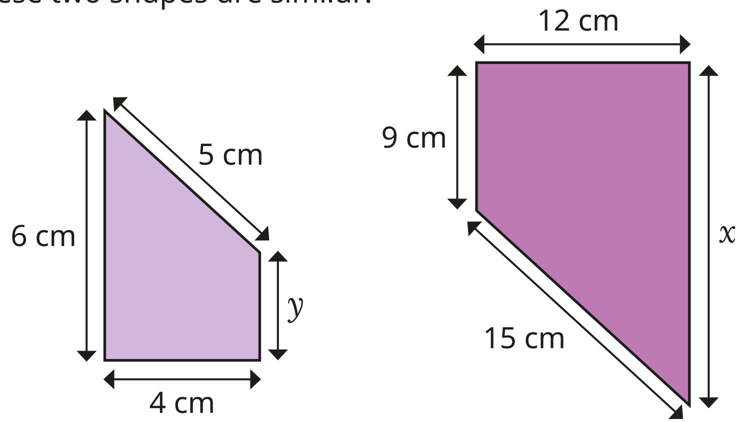
  - Find the lengths of b and c.
  - Measure the sizes of all the angles.

  What do you notice?

- Which of the shapes are similar to shape A?



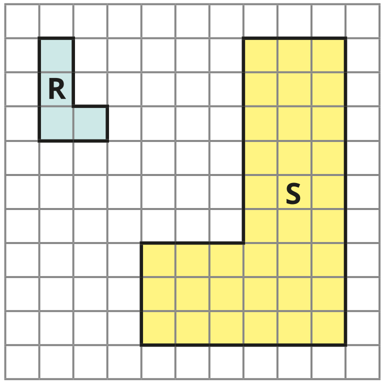
- These two shapes are similar.

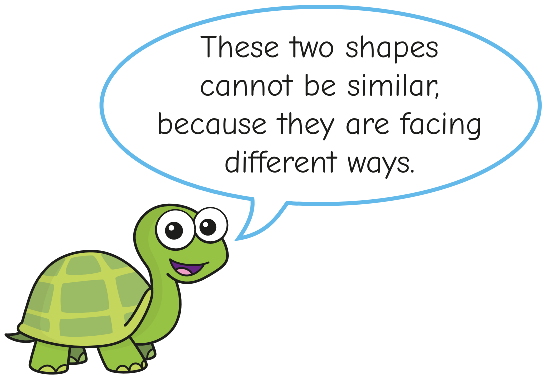


Find the lengths of x and y.

Similar shapes

Reasoning and problem solving





Do you agree with Tiny?

Explain your answer.

No

The Eiffel Tower is 320 m tall and 120 m wide.

Tommy makes a scale model of the Eiffel Tower.

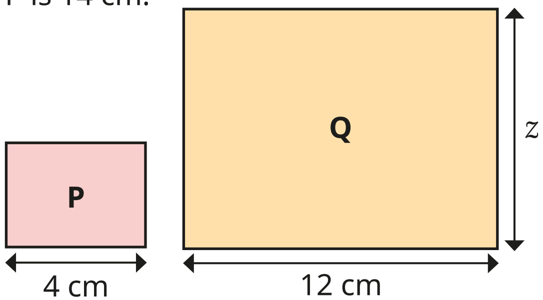
His model is 16 cm tall.

How wide is his model?

6 cm

Rectangles P and Q are similar.

The perimeter of rectangle P is 14 cm.



Work out length z.

z = 9 cm

Ratio problems

Notes and guidance

In this small step, children use what they have learnt so far in this block to solve a variety of problems involving ratio.

Children use representations from earlier steps to help them see the multiplicative relationships between ratios. They recognise that when they multiply or divide from one amount to another, they do the same for the other value to keep the ratios equivalent. Children may see that this method is similar to finding equivalent fractions. When using double number lines, children can explore the vertical as well as horizontal multiplicative relationships.

Representing problems using bar models supports the interpretation of word ratio problems. These models can be used for a wide range of question types, such as: "If there are ______ blue/red/total, how many blue/red/total are there?" and "If there are ______ more red than blue, how many blue/red/total are there?"

Things to look out for

- Children may confuse the "total" amount for the value of a missing part.
- Children may use additive rather than multiplicative relationships.

Key questions

- What is the ratio of ______ to ______?
- If there are ______, how many ______ must there be?
- If the total number of ______ is ______, how many ______ must there be?
- If there are ______ more ______ than ______, how many are there in total?
- How can you draw a bar model to solve the problem? Which parts of the model do you know? How can you work out the remaining parts?

Possible sentence stems

- The ratio of ______ to ______ is ______ : ______
- I know that ______ multiplied/divided by ______ is equal to ______, so to find out how many ______ there are, I need to multiply/divide by ______

National Curriculum links

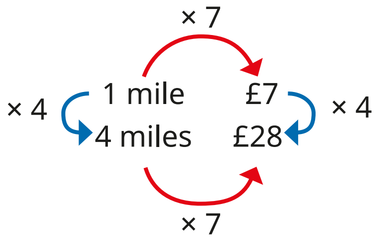
- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

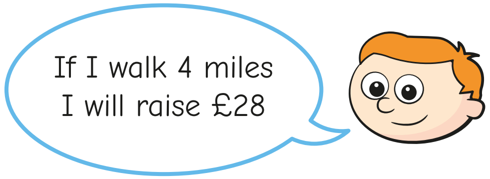
Ratio problems

Key learning

- Ron is doing a sponsored walk for charity.

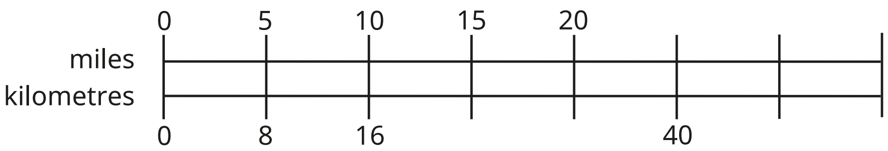
  For every mile he walks, he will raise £7





  - How much will Ron raise if he walks 3 miles?
  - How much will Ron raise if he walks 22 miles?
  - How many miles will Ron need to walk to raise £42?

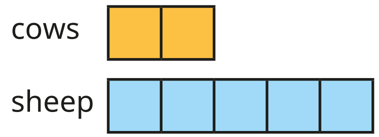
- The double number line shows the relationship between miles and kilometres.
  - Complete the double number line.



  - Complete the statements.

  55 miles = ______ km ______ miles = 96 km

- On a farm, for every 2 cows, there are 5 sheep.



  Use bar models to answer the questions.
  - If there are 4 cows, how many animals are there altogether?
  - If there are 35 animals altogether, how many cows are there?
  - If there are 50 sheep, how many cows are there?
  - If there are 12 cows, how many more sheep are there than cows?

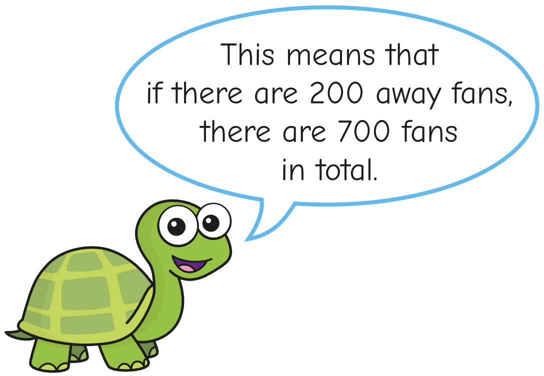
- In a car park, there are 4 blue cars for every 7 red cars.
  - If there are 20 blue cars, how many red cars are there?
  - If there are 28 red cars, how many blue cars are there?
  - If there are 22 cars in total, how many of them are blue?
  - If there are 12 blue cars, how many more red cars are there than blue cars?
  - If there are 30 more red cars than blue cars, how many cars are there in total?

Ratio problems

Reasoning and problem solving

At a football match, the ratio of home fans to away fans is 7 : 2

Home fans	Away fans
7	2
14	4
21	6
28	8

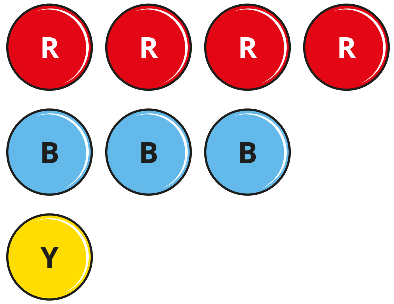


Do you agree with Tiny?

Explain your answer.

No

The ratio of red to blue to yellow counters is 4 : 3 : 1



If there are 148 red counters, how many yellow counters are there?

If there are 50 more blue counters than yellow counters, how many red counters are there?

If there are 608 counters in total, how many of them are red?

How did you work this out?

Compare answers with a partner.

37

100

304

Proportion problems

Notes and guidance

In this small step, children explore different strategies for solving proportion problems.

Building on previous steps, a double number line is a useful representation for these types of problems. Begin by looking at simple one-step problems that involve a single multiplication or division, for example "4 ______ cost ______. What do 12 cost?" or "4 ______ cost ______. What do 2 cost?"

Then move on to two-step problems, where children first need to find the value of 1 ______ through division. Again, seeing this on a double number line helps to show children that both values need to be divided by the same amount to find 1, then both new values can be multiplied by the same amount to find any new value.

Things to look out for

- In one-step proportion problems, children may multiply by the wrong amount or add instead of multiply.
- When using a double number line in two-step proportion problems, children may count the step to zero and divide by the wrong amount.

Key questions

- What is the multiplicative relationship between ______ and ______?
- If 3 ______ cost £______, how much do 12 ______ cost?
- If 5 ______ cost £______, how can you work out what 1 ______ costs?
- Once you know what 1 ______ costs, how can you work out what 8 ______ cost?
- How can a double number line help you solve this proportion problem?

Possible sentence stems

- If ______ costs ______, then ______ costs ______
- To get from ______ to ______, I multiply/divide by ______
- To find the cost of 1 ______, I will …

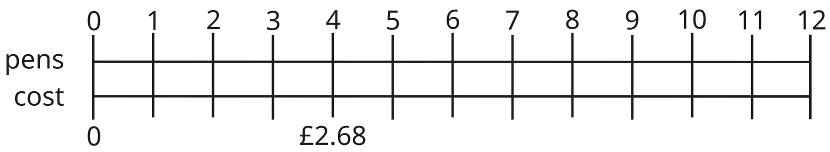
National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Proportion problems

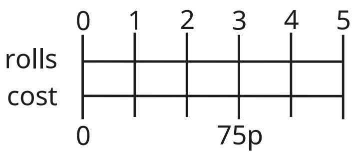
Key learning

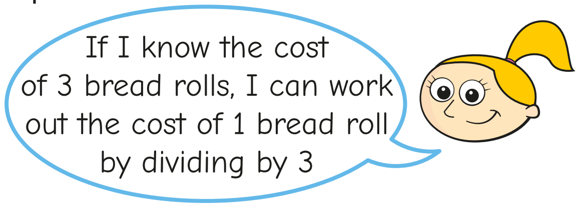
- 4 pens cost £2.68



  - Use the double number line to work out the cost of 12 pens.
  - Use a double number line to help you work out the cost of buying:
    - 36 pens
    - 360 pens
  - Use a double number line to help you work out how many pens can be bought for:
    - £1.34
    - £26.80

- Eva buys 3 bread rolls for 75p.



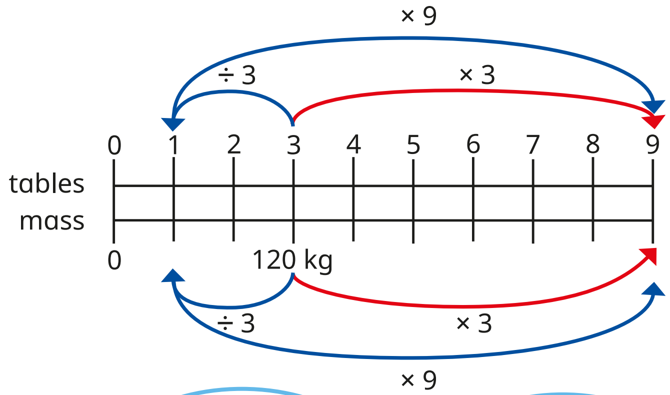


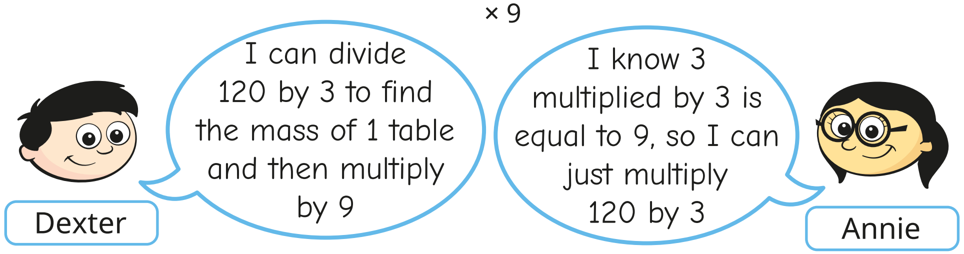
Tell a partner how this will help Eva to find the cost of 5 bread rolls.

What is the cost of 5 bread rolls?

- 3 tables have a total mass of 120 kg.

Dexter and Annie are working out the mass of 9 tables.





Use both methods to find the mass of 9 tables.

Whose method do you prefer?

- A shop sells flour at the price of 54p for 0.3 kg.

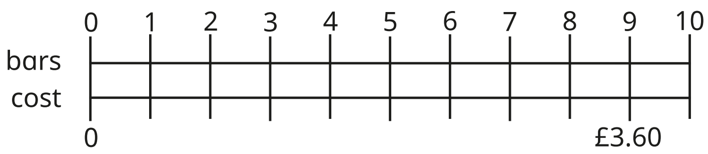
How much would it cost to buy these masses of flour?

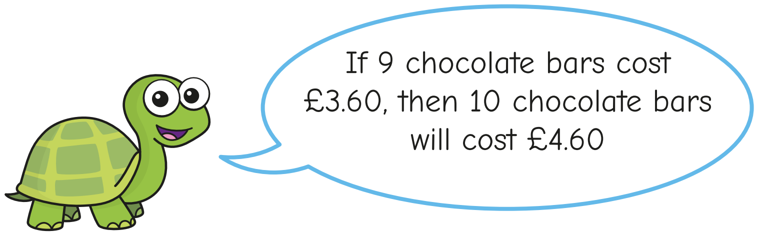
150 g	700 g	2 kg	5.2 kg

Proportion problems

Reasoning and problem solving

The cost of 9 chocolate bars is £3.60





Do you agree with Tiny?

Explain your answer.

No

Tiny has added £1, but each chocolate bar does not cost £1

1 chocolate bar costs £3.60 ÷ 9 = 40p

10 chocolate bars cost 40p × 10 = £4

It costs a company 12p to make 10 marbles.

Marbles are sold in boxes of 500 for £6.50

How much profit does the company make on every box of marbles?

How did you work it out?

50p

A car travelling at a constant speed travels 24 km in 12 minutes.

How far will the car travel in 1 hour?

How long will it take the car to travel 84 km?

How did you work it out?

120 km

42 minutes

Recipes

Notes and guidance

For this small step, children apply their knowledge of ratio and proportion to solving problems involving ingredients for recipes.

As a class, look at a simple list of ingredients for, for example, 4 people and discuss how it could be adapted for 8/2/40 people. After solving simple scaling-up/scaling-down problems, children look at problems with a given amount of a specific ingredient, for example "The recipe needs 100 g of butter. Aisha has 500 g of butter. How much ______ can she make?"

Children can then explore multi-step problems that involve multiplying and dividing quantities of ingredients, for example adjusting the quantities for 4 people to 5 people by dividing each ingredient by 4 and then multiplying by 5

Things to look out for

- Children may only scale one of the ingredients instead of all of them.
- Children may not see efficient methods for two-step problems.
- Children may make errors when they need to convert between units of measure.

Key questions

- How can a double number line help you decide how much of each ingredient you need?
- How many times more people are there? How will this affect the amount of each ingredient needed?
- Do you need to find the amounts needed for one person first? Why or why not?
- What is the greatest number of ______ you can make with ______?
- How does changing the quantities in a recipe link to using scale factors?

Possible sentence stems

- There are ______ times as many people, so I need ______ times as much of each ingredient.
- First, I will find the quantities for 1 person by dividing by ______ and then I will multiply this by ______

National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Recipes

Key learning

- Here are some ingredients for cupcakes.

  Tom wants to make 10 cupcakes.

  Explain to a partner how to work out what ingredients Tom will need.

  How much of each ingredient will Tom need to make the different numbers of cupcakes?

  Cupcakes (makes 5)

  100 g flour

  2 eggs

  40 g sugar

  15 cupcakes | 20 cupcakes | 25 cupcakes

- Here are some ingredients for soup.

  How much of each ingredient is needed to make soup for the different numbers of people?

  Soup (for 6 people)

  1 onion

  60 g butter

  180 g lentils

  1.2 litres stock

  480 ml tomato juice

  2 people | 1 person | 9 people

- Sam is making pancakes.

  She follows a recipe with this list of ingredients.

  She has 1.2 litres of milk and wants to make as many pancakes as she can.

  How many eggs will she need?

  Pancakes

  120 g plain flour

  2 eggs

  300 ml milk

- Here are the ingredients for an apple crumble.

  How much of each ingredient is needed to make apple crumble for the different numbers of people?

  Apple crumble (5 people)

  300 g plain flour

  225 g brown sugar

  200 g butter

  450 g apples

  10 people | 12 people

- A baker uses 12 eggs to make 108 muffins.

  How many muffins will 20 eggs make?

  How many different ways can you work it out?

Recipes

Reasoning and problem solving

Here are the ingredients for 10 flapjacks.

Flapjacks (makes 10)

120 g butter

100 g brown sugar

4 tablespoons golden syrup

250 g oats

40 g sultanas

Huan has 180 g butter.

What is the greatest number of flapjacks he can make?

How much of each of the other ingredients will he need?

15

150 g brown sugar

6 tablespoons golden syrup

375 g oats

60 g sultanas

Here are the ingredients for making one smoothie.

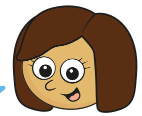
Smoothie

2 apples

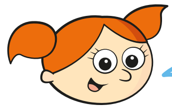
3 bananas

500 ml milk

I have 7 apples, 9 bananas and 1 litre of milk.



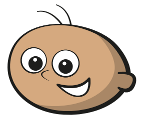
Kim



Alex

I have 6 apples, 10 bananas and 1.5 litres of milk.

I have 10 apples, 5 bananas and 750 ml of milk.



Tommy

Who can make the most smoothies?

Alex